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(MATHEMATICS VALUES CHASE YEAR 01-10-2015 to 30-09-2016)

VEDIC MATHEMATICS
&
MODERN MATHEMATICS

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COURSE 05 PART – 2
CREATOR SPACE
(4-SPACE)
Fourth Week : Day 4

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Let us first revisit MA / M. Sc (mathematics courses of different universities of India and top universities of the world

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Kurushetra University, Kurushetra, India

Scheme of Examination for M.Sc. Mathematics
w.e.f 2011-12

Semester – I

MM-401 Advanced Abstract Algebra
MM-402 Real Analysis
MM-403 Topology
MM-404 Complex Analysis – I
MM-405 Differential Equations – I
MM-406 Practical-I

Semester – II
MM-407 Advanced Abstract Algebra – II
MM-408 Real Analysis – II
MM-409 Computer Programming (Theory)
MM-410 Complex Analysis – II
MM-411 Differential Equations – II
MM-412 Practical-II

Semester – III

MM-501 Functional Analysis
MM-502 Analytical Mechanics and Calculus of Variations

MM-503
(Opt. (i))
Elasticity 80 20 100 3 Hours

MM-503
(Opt. (ii)
Difference Equations-I

MM-503
(Opt. (iii)
Analytic Number Theory
MM-503
(Opt. (iv)

Number Theory
MM-504

(Opt. (i)
Fluid Mechanics – I 80 20 100 3 Hours
MM-504
(Opt. (ii)
Mathematical Statistics 80 20 100 3 Hours
MM-504
(Opt. (iii)
Algebraic Coding Theory 80 20 100 3 Hours
MM-504
(Opt. (iv)
Commutative Algebra 80 20 100 3 Hours

MM-505
(Opt. (i)
Integral Equations 80 20 100 3 Hours
MM-505
(Opt. (ii)
Mathematical Modeling 80 20 100 3 Hours
MM-505
(Opt. (iii)
Linear Programming 80 20 100 3 Hours
MM-505
(Opt. (iv)
Fuzzy Sets &
Applications –I
80 20 100 3 Hours
MM-506 Practical-III --
100
4 Hours

Semester – IV

MM-507 General Measure and
Integration Theory
MM-508 Partial Differential Equations

MM-509
(Opt. (i)
Mechanics of Solids
MM-509
(Opt. (ii)
Difference Equations-II
MM-509
(Opt. (iii)
Algebraic Number
Theory
MM-509
(Opt. (iv)
Mathematics for Finance
& Insurance
MM-510
(Opt. (i)
Fluid Mechanics-II
MM-510
(Opt. (ii)
Boundary Value
Problems
MM-510
(Opt. (iii)
Non-Commutative Rings
MM-510
(Opt. (iv)
Advanced Discrete
Mathematics
MM-511
(Opt. (i)
Mathematical Aspects of
Seismology
MM-511
(Opt. (ii)
Dynamical Systems

MM-511
(Opt. (iii)
Operational Research
MM-511
(Opt. (iv)
Fuzzy Sets &
Applications-II
MM-512 Practical-IV --
100
4 Hours

Semester – I

MM-401: Advanced Abstract Algebra-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Questions)

Automorphisms and Inner automorphisms of a group G. The groups Aut(G) and Inn(G). Automorphism group of a cyclic group. Normalizer and Centralizer of a non-empty subset of a group G. Conjugate elements and conjugacy classes. Class equation of a finite group G and its applications. Derived group (or a commutator subgroup) of a group G. perfect groups. Zassenhau's Lemma. Normal and Composition series of a group G. Scheier's refinement theorem. Jordan Holder theorem. Composition series of groups of order pn and of Abelian groups. Caunchy theorem for finite groups. . -groups and p-groups. Sylow .-subgroups and Sylow p-subgroups. Sylow's Ist, IInd and IIIrd theorems. Application of Sylow theory to groups of smaller orders.

Section – II (Two Questions)

Characteristic of a ring with unity. Prime fields Z/pZ and Q. Field extensions. Degree of an extension. Algebraic and transcendental elements. Simple field extensions. Minimal polynomial of an algebraic element. Conjugate elements. Algebraic extensions. Finitely generated algebraic extensions. Algebraic closure and algebraically closed fields. Splitting fields., finite fields.. Normal extensions.

Section – III (Two Questions)

Separable elements, separable polynomials and separable extensions. Theorem of primitive element. Perfect fields. Galois extensions. Galois group of an extension. Dedekind lemma Fundamental theorem of Galois theory. Frobenius automorphism of a finite field. Klein's 4-group and Diheadral group. Galois groups of polynomials. Fundamental theorem of Algebra.

Section – IV (Two Questions)

Solvable groups Derived series of a group G. Simplicity of the Alternating group An (n>5). Non-solvability of the symmetric group Sn and the Alternating group An (n>5).
Roots of unity Cyclotomic polynomials and their irreducibility over Q Radicals extensions. Galois radical extensions. Cyclic extensions. Solvability of polynomials by radicals over Q. Symmetric functions and elementary symmetric functions. Construction with ruler and compass only.

Recommended Books:

1. I.D. Macdonald. :The theory of Groups
2. P.B. Bhattacharya

S.K. Jain & S.R. Nagpal
: Basic Abstract Algebra (Cambridge University
Press 1995)

Reference Books:

1. Vivek Sahai and Vikas Bist :
Algebra (Narosa publication House)
2. I.S. Luthar and I.B.S. Passi
: Algebra Vol. 1 Groups (Narosa publication House)
3. I.N. Herstein :
Topics in Algebra (Wiley Eastern Ltd.)
4. Surjit Singh and Quazi Zameeruddin :
Modern Algebra (Vikas Publishing House 1990)

Semester-I

MM-402 : REAL ANALYSIS –I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and existence of Riemann Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, integration of vector-valued functions, Rectifiable curves.
(Scope as in Chapter 6 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Section-II (Two Questions)

Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weirstrass M-test, Abel's test and Dirichlet's test for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann Stieltjes integration, uniform convergence and differentiation, existence of a real continuous nowhere differentiable function, equicontinous families of functions, Weierstrass approximation theorem (Scope as in Sections 7.1 to 7.27 of Chapter 7 of Principles of Mathematical Analysis by Walter Rudin, Third Edition).

Section-III (Two Questions)

Functions of several variables : linear transformations, Derivative in an open subset of Rn, Chain rule, Partial derivatives, directional derivatives, the contraction principle, inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Derivatives of higher order, mean value theorem for real functions of two variables, interchange of the order of differentiation, Differentiation of integrals.

(Scope as in relevant portions of Chapter 9 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition)

Section-IV (Two Questions)

Power Series : Uniqueness theorem for power series, Abel's and Tauber's theorem, Taylor's theorem, Exponential & Logarithm functions, Trigonometric functions, Fourier series, Gamma function
(Scope as in Chapter 8 of 'Principles of Mathematical Analysis' by Walter Rudin, Third
Edition).

Integration of differential forms: Partitions of unity, differential forms, stokes theorem

(scope as in relevant portions of Chapter 9 & 10 of 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition).

Recommended Text:

'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.

Reference Books :

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.
4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
5.Serge Lang, Analysis I & II, Addison-Wesley Publishing Company Inc., 1969.

M.Sc.(P)Mathematics Semester-I

MM-403: TOPOLOGY

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set , closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense sets. A characterization of dense sets.
Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology.
Relative (induced) Topology and subspace of a topological space. Alternate methods of

defining a topology using 'properties' of 'Neighbourhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator and 'base'.
First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorem.
Comparison of Topologies on a set, about intersection and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice (scope as in theorems 1-16, chapter 1 of Kelley's book given at Sr. No. 1).

SECTION-II (Two Questions)

Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding.
Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness, Characterisation of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces.

T0 , T1, T2,Regular and T3 separation axioms, their characterization and basic properties i.e. hereditary property of T0 , T1, T2, Regular and T3 spaces, and productive property of T1 and T2 spaces.
Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space (scope as in theorems 1, 2, 3, 5, 6, 8-11, Chapter 3 and relevant portion of chapter 4 of Kelley's book given at Sr.No.1)

Section-III (Two Questions)

Completely regular and Tychonoff (T 3 1/2), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem.
Normal and T4 spaces : Definition and simple examples, Urysohn's Lemma, complete regularity of a regular normal space, T4 implies Tychonoff, Tietze's extension theorem (Statement only). (Scope as in theorems 1-7, Chapter 4 of Kelley's book given at Sr. No. 1).

Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters, Ultra filter (u.f.) and its characterizations, Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.
Convergence of filters: Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters. Hausdorffness and filter convergence.

Section-IV (Two Questions)

Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), continuity and compact sets, compactness and separation properties,

Closedness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space.
Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse, compactification and Hausdorff compactification, Stone-Cech compactification, (Scope as in theorems 1,7-11, 13, 14, 15, 22-24, Chapter 5 of Kelley's book given at Sr. No. 1).

Books :

1. Kelley, J.L. : General Topology.
2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

Semester-I

MM-404: COMPLEX ANALYSIS-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Power series, its convergence, radius of convergence, examples, sum and product, differentiability of sum function of power series, property of a differentiable function with derivative zero. expz and its properties, logz, power of a complex number (z), their branches with analyticity.
Path in a region, smooth path, p.w. smooth path, contour, simply connected region, multiply connected region, bounded variation, total variation, complex integration,
Cauchy-Goursat theorem, Cauchy theorem for simply and multiply connected domains.

Section II (Two Questions)

Index or winding number of a closed curve with simple properties. Cauchy integral formula. Extension of Cauchy integral formula for multiple connected domain. Higher order derivative of Cauchy integral formula. Gauss mean value theorem Morera's theorem. Cauchy's inequality. Zeros of an analytic function, entire function, radius of convergence of an entire function, Liouville's theorem, Fundamental theorem of algebra, Taylor's theorem.

Section-III (Two Questions)

Maximum modulus principle, Minimum modulus principle. Schwarz Lemma.
Singularity, their classification, pole of a function and its order. Laurent series, Cassorati

– Weiertrass theorem Meromorphic functions, Poles and zeros of Meromorphic functions. The argument principle, Rouche's theorem, inverse function theorem.
Section-IV (Two Questions)

Residue : Residue at a singularity, residue at a simple pole, residue at infinity. Cauchy residue theorem and its use to calculate certain integrals, definite integral (ò0
2. f(cos.,
sin.) d., ò-8
¥f(x)dx), integral of the type ò0
¥f(x) sinmx dx or ò0
¥ f(x) cosmx dx, poles on the real axis, integral of many valued functions.

Bilinear transformation, their properties and classification, cross ration, preservance of cross ration under bilinear transformation, preservance of circle and straight line under bilinear transformation, fixed point bilinear transformation, normal form of a bilinear transformation. Definition and examples of conformal mapping, critical points.

Books recommended :

1.
Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2.
Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3.
Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1.
Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2.
Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3.
D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4.
Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5.
E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6.
S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-I

MM-405: Differential Equations –I
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section –I (Two Questions)

Preliminaries: Initial value problem and equivalent integral equation, e-approximate solution, equicontinuous set of functions.
Basic theorems: Ascoli-Arzela theorem, Cauchy –Peano existence theorem and its corollary. Lipschitz condition. Differential inequalities and uniqueness, Gronwall's inequality. Successive approximations. Picard-Lindelöf theorem. Continuation of

solution, Maximal interval of existence, Extension theorem. Kneser's theorem (statement only)
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-II (Two Questions)

Linear differential systems: Definitions and notations. Linear homogeneous systems; Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Nonhomogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-III (Two Questions)

Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's Identity, Fundamental set, More Wronskian theory. Reduction of order. Non-homogeneous LDE. Variation of parameters. Adjoint equations, Lagrange's Identity, Green's formula. Linear equation of order n with constant coefficients. (Relevant portions from the books of 'Theory of Ordinary Differential Equations' by Coddington and Levinson and the book 'Differential Equations' by S.L. Ross)

Section –IV (Two Questions)

System of differential equations, the n-th order equation. Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness theorems: Kamke's theorem, Nagumo's theorem and Osgood theorem.
(Relevant portions from the book 'Ordinary Differential Equations' by P. Hartman)

Referneces:

1.
E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations,
Tata McGraw-Hill , 2000.
2.
S.L. Ross, Differential Equations, John Wiley & Sons,
3.
P. Hartman, Ordinary Differential Equations, John Wiley & Sons NY, 1971.
4.
G. Birkhoff and G.C. Rota, Ordinary Differential Equations, John Wiley & Sons, 1978.
5.
G.F. Simmons, Differential Equations, Tata McGraw-Hill , 1993.
6.
I.G. Petrovski, Ordinary Differential Equations, Prentice-Hall, 1966.
7.
D. Somasundaram, Ordinary Differential Equations, A first Course, Narosa Pub., 2001.
8.

S.G. Deo, V. Lakshmikantham and V. Raghavendra, Textbook of Ordinary Differential Equations, Tata McGraw-Hill , 2006.

Semester-I

Paper MM-406 : Practical-I

Examination Hours : 4 hours
Max. Marks : 100
Part-A : Problem Solving

In this part, problem-solving techniques based on papers MM-401 to MM-405 will be taught.

Part-B : Implementation of the following programs in ANSI C.

1.
Use of nested if.. .else in finding the smallest of four numbers.
2.
Use series sum to compute sin(x) and cos(x) for given angle x in degrees.
Then, check error in verifying sin2 x+cos2 (x)=1.
3.
Verify Sn3 ={Sn }2, (where n=1,2,..,m) & check that prefix and postfix increment operator gives the same result.
4.
Compute simple interest of a given amount for the annual rate = .12 if amount >=10,000/-or time >=5 years; =.15 if amount >=10,000/-and time >=5 years; and = .10 otherwise.
5.
Use array of pointers for alphabetic sorting of given list of English words.
6.
Program for interchange of two rows or two columns of a matrix. Read/write input/output matrix from/to a file.
7.
Calculate the eigenvalues and eigenvectors of a given symmetric matrix of order 3.
8.
Calculate standard deviation for a set of values {x(j)j=l,2,...,n} having the corresponding frequencies {f(j)j=l,2,...,n}.
9.
Find GCD of two positive integer values using pointer to a pointer.
10.
Compute GCD of 2 positive integer values using recursion.
11.
Check a given square matrix for its positive definite form.
12.
To find the inverse of a given non-singular square matrix.

Note :-Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee’s performance in written solutions/programs, execution of computer programs and viva-voce examination.

Semester – II

MM-407: Advanced Abstract Algebra-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Commutators and higher commutators. Commutators identities. Commutator subgroups.
Derived group. Three subgroups Lemma of P.Hall. Central series of a group G. Nilpotent groups. Centre of a nilpotent group. Subgroups and factor subgroups of nilpotent groups.
Finite nilpotent groups. Upper and lower central series of a group G and their properties.
Subgroups of finitely generated nilpotent groups. Sylow-subgroups of nilpotent groups. (Scope of the course as given in the book at Sr. No. 2).

Section-II (Two Questions)

Similar linear transformations. Invariant subspaces of vector spaces. Reduction of a linear transformation to triangular form. Nilpotent transformations. Index of nilpotency of a nilpotent transformation. Cyclic subspace with respect to a nilpotent transformation.
Uniqueness of the invariants of a nilpotent transformation.

Primary decomposition theorem. Jordan blocks and Jordan canonical forms. Cyclic module relative to a linear transformation. Companion matrix of a polynomial f(x).
Rational Canonicals form of a linear transformation and its elementary divisior.
Uniqueness of the elementary divisior. (Sections 6.4 to 6.7 of the book. Topics in Algebra by I.N. Herstein).

Section-III (Two Questions)

Modules, submodules and quotient modules. Module generated by a non-empty subset of an R-module. Finitely generated modules and cyclic modules. Idempotents.
Homomorphism of R-modules. Fundamental theorem of homomorphism of R-modules.
Direct sum of modules. Endomorphism rings EndZ(M) and EndR(M) of a left R-module

M. Simple modules and completely reducible modules (semi-simple modules). Finitely generated free modules. Rank of a finitely generated free module. Submodules of free modules of finite rank over a PID. (Sections 14.1 to 14.5 of the book Basic Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal)

Section-IV (Two Questions)

Endomorphism ring of a finite direct sum of modules. Finitely generated modules.
Ascending and descending chains of sub modules of an R-module. Ascending and Descending change conditions (A.C.C. and D.C.C.). Noetherian modules and Noetherian rings. Finitely co-generated modules. Artinian modules and Artinian rings. Nil and nilpotent ideals. Hilbert Basis Theorem. Structure theorem of finite Boolean rings.
Wedeerburn-Artin theorem and its consequences. (sections 19.1 to 19.3 of the book Basic
Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal).

Recommended Books:

1. Basic Abstract Algebra : P.B. Bhattacharya S.R. Jain and S.R. Nagpal
2. Theory of Groups : I.D. Macdonald
3. Topics in Algebra : I.N. Herstein
4. Group Theory : W.R. Scott

Semester-II

MM-408 : REAL ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Lebesgue outer measure, elementary properties of outer measure, Measurable sets and
their properties, Lebesgue measure of sets of real numbers, algebra of measurable sets,
Borel sets and their measurability, characterization of measurable sets in terms of open,
closed, F and G sets, existence of a non-measurable set.

Lebesgue measurable functions and their properties, characteristic functions, simple
functions, approximation of measurable functions by sequences of simple functions,
measurable functions as nearly continuous functions. Borel measurability of a function.

Section-II (Two Questions)

Almost uniform convergence, Egoroff's theorem, Lusin's theorem, convergence in
measure, F.Riesz theorem that every sequence which is convergent in measure has an
almost everywhere convergent subsequence.

The Lebesgue Integral :
Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of
finite measure and its properties, Lebsegue integral as a generalization of the Riemann
integral, Bounded convergence theorem, Lebesgue theorem regarding points of
discontinuities of Riemann integrable functions.

Section-III (Two Questions)

Integral of a non negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral, Lebesgue convergence theorem.
Differentiation and Integration :
Differentiation of monotone functions, Vitali's covering lemma, the four Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions.

Section-IV (Two Questions)

Differentiation of an integral, absolutely continuous functions, convex functions, Jensen's inequality.

The Lp spaces

The Lp spaces, Minkowski and Holder inequalities, completeness of Lp spaces, Bounded linear functionals on the Lp spaces, Riesz representation theorem.

Recommeded Text :

'Real Analysis' by H.L.Royden (3rd Edition) Prentice Hall of India, 1999.

Reference Books :

1.
G.de Barra, Measure theory and integration, Willey Eastern Ltd.,1981.
2.
P.R.Halmos, Measure Theory, Van Nostrans, Princeton, 1950.
3.
I.P.Natanson, Theory of functions of a real variable, Vol. I, Frederick Ungar Publishing Co., 1961.
4.
R.G.Bartle, The elements of integration, John Wiley & Sons, Inc.New York, 1966.
5.
K.R.Parthsarthy, Introduction to Probability and measure, Macmillan Company of India Ltd.,Delhi, 1977.
P.K.Jain and V.P.Gupta, Lebesgue measure and integration, New age International (P) Ltd., Publishers, New Delhi, 1986.

Semester-II

MM-409 : Computer Programming (Theory)

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Numerical constants and variables; arithmetic expressions; input/output; conditional flow; looping.

Section-II (Two Questions)

Logical expressions and control flow; functions; subroutines; arrays.

Section- III(Two Questions)

Format specifications; strings; array arguments, derived data types.

Section- IV(Two Questions)

Processing files; pointers; modules; FORTRAN 90 features; FORTRAN 95 features.

Recommended Text:

V. Rajaraman : Computer Programming in FORTRAN 90 and 95; Printice-Hall of India Pvt. Ltd., New Delhi, 1997.

References :

1.
V. Rajaraman : Computer Programming in FORTRAN 77, Printice-Hall of India
Pvt. Ltd., New Delhi, 1984.
2.
J.F. Kerrigan : Migrating of FORTRAN 90, Orielly Associates, CA, USA, 1993.
3.
M.Metcalf and J.Reid : FORTRAN 90/95 Explained, OUP, Oxford, UK, 1996.

Semester-II

MM-410 : COMPLEX ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Spaces of analytic functions and their completeness, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, infinite products, Weierstrass factorization theorem, Factorization of sine function, Gamma function and its properties, functional equation for
gamma function, Integral version of gamma function.

Section- II (Two Questions)

Reimann-zeta function, Riemann's functional equation, Runge's theorem, MittagLeffler's theorem.
Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation , Schwarz reflection principle.

Section –III (Two Questions)

Monodromy theorem and its consequences. Harmonic function as a disk, Poisson's
Kernel. Harnack's inequality, Harnack's theorem, Canonical product, Jensen's formula,
Poisson-Jensen formula, Hadamard's three circle theorem. Dirichlet problem for a unit disk. Dirichlet problem for a region, Green's function.

Section –IV (Two Questions)

Order of an entire function, Exponent of convergence, Borel theorem, Hadamard's factorization theorem. The range of an analytic function, Bloch's theorem, Little-Picard theorem, Schottky's theorem, Montel-Carathedory theorem, Great Picard theorem. Univalent functions, Bieberbach's conjecture (Statement only), and 17/ 4 theorem.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.

2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.

3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.

2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.

3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.

4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.

5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.

6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-II

MM-411: DIFFERENTIAL EQUATIONS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all, taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section –I (Two Questions)

Linear second order equations: Preliminaries, self adjoint equation of second order, Basic facts, superposition principle, Riccati's equation, Prüffer transformation, zero of a solution, Oscillatory and non-oscillatory equations. Abel's formula. Common zeros of solutions and their linear dependence.

(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book
'Textbook of Ordinary Differential Equations' by Deo et al.)

Section –II (Two Questions)

Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and its
corollaries. Elementary linear oscillations.
Autonomous systems: the phase plane, paths and critical points, Types of critical points;
Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book
'Textbook of Ordinary Differential Equations' by Deo et al.)

Section-III (Two Questions)

Critical points and paths of non-linear systems: basic theorems and their applications.
Liapunov function. Liapunov's direct method for stability of critical points of non-linear systems.
Limit cycles and periodic solutions: Limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence criterion. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. Index of a critical point.

(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-IV (Two Questions)

Second order boundary value problems(BVP): Linear problems; periodic boundary
conditions, regular linear BVP, singular linear BVP; non-linear BVP. Sturm-Liouville BVP: definitions, eigen value and eigen function. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values. Green's function.
Applications of boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.
(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Referneces:

1.
E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations,
Tata McGraw-Hill , 2000.
2.
S.L. Ross, Differential Equations, John Wiley & Sons,
3.
S.G. Deo, V. Lakshmikantham and V. Raghavendra, Textbook of Ordinary Differential Equations, Tata McGraw-Hill , 2006.
4.
P. Hartman, Ordinary Differential Equations, John Wiley & Sons NY, 1971.
5.
G. Birkhoff and G.C. Rota, Ordinary Differential Equations, John Wiley & Sons, 1978.

6.
G.F. Simmons, Differential Equations, Tata McGraw-Hill , 1993.
7.
I.G. Petrovski, Ordinary Differential Equations, Prentice-Hall, 1966.
8.
D. Somasundaram, Ordinary Differential Equations, A first Course, Narosa Pub., 2001.

Semester-II

Paper MM-412 : Practical-II

Examination Hours : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-407 to MM-411 will be taught.

Part-B : Implementation of the following programs in FORTRAN-90

1.
Calculate the area of a triangle with given lengths of its sides.
2.
Given the centre and a point on the boundary of a circle, find its perimeter and area.
3.
To check an equation ax2+ by2+2cx+2dy+e=0 in (x, y) plane with given coefficients for representing parabola/ hyperbola/ ellipse/ circle or else.
4.
For two given values x and y, verify g*g=a*h, where a, g and h denote the arithmetic, geometric and harmonic means respectively.
5.
Use IF..THEN…ELSE to find the largest among three given real values.
6.
To solve a quadratic equation with given coefficients, without using COMPLEX data type.
7.
To find the location of a given point (x,y) i) at origin, ii) on x-axis or y-axis iii) in quadrant I, II, III or IV.
8.
To find if a given 4-digit year is a leap year or not.
9.
To find the greatest common divisor (gcd) of two given positive integers.
10. To verify that sum of cubes of first m positive integers is same as the square of the sum of these integers.
11. Find error in verifying sin(x+y)= sin(x) cos(y)+cos(x)sin(y), by approximating the sin(x) and cos(x) functions from the finite number of terms in their series expansions.
12. Use SELECT…CASE to calculate the income tax on a given income at the existing rates.

Note :-Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

SEMESTER-III

MM-501 Functional Analysis

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Normed linear spaces, Banach spaces and examples, subspace of a Banach space, completion of a normed space, quotient space of a normed linear space and its completeness, product of normed spaces, finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, F.Riesz's lemma.
Bounded and continuous linear operators, differentiation operator, integral operator, bounded linear extension, linear functionals, bounded linear functionals, continuity and boundedness, definite integral, canonical mapping, linear operators and functionals on finite dimensional spaces, normed spaces of operators, dual spaces with examples. (Scope of this section is as in relevant parts of Chapter 2 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

SECTION-II (Two Questions)

Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces, application to bounded linear functionals on C[a,b], Riesz-representation theorem for bounded linear functionals on C[a,b], adjoint operator, norm of the adjoint operator.
Reflexive spaces, uniform boundedness theorem and some of its applications to the space of polynomials and fourier series. (Scope of this section is as in relevant parts of sections 4.1 to 4.7 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

SECTION-III (Two Questions)

Strong and weak convergence, weak convergence in lp , convergence of sequences of operators, uniform operator convergence, strong operator convergence, weal operator convergence, strong and weak* convergence of a sequence of functionals. Open mapping theorem, bounded inverse theorem, closed linear operators, closed graph theorem, differential operator, relation between closedness and boundedness of a linear operator. (Scope of this section is as in relevant parts of sections 4.8, 4.9, 4.12 and 4.13 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)
Inner product spaces, Hilbert spaces and their examples, pythagorean theorem, Apolloniu's identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct

sums, projection theorem, characterization of sets in Hilbert spaces whose space is dense.
(Scope as in relevant parts of sections 3.1, 3.2 and 3.3 of Chapter 3 of ‘Introductory Functional Analysis with Applications’ by E.Kreyszig)

SECTION-IV (Two Questions)

Orthonormal sets and sequences, Bessel’s inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval’s identity,
separable Hilbert spaces.Representation of functionals on Hilbert spaces, Riesz representation theorem for bounded linear functionals on a Hilbert space, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space.
Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self adjoint, unitary, normal, positive and projection operators. (Scope of this
section is as in relevant parts of sections 3.4 to 3.6 and 3.8 to 3.10 of Chapter 3 and sections 9.3 to 9.6 of Chapter 9 of ‘Introductory Functional Analysis with Applications’
by E.Kreyszig.

Recommended Text:

E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1978.

References:

1.
G.F.Simmons: Introduction to Topology and Modern Analysis, McGraw Hill Book Co.,New York, 1963.
2.
C.Goffman and G.Pedrick: First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
3.
G.Bachman and L.Narici, Functional Analysis, Academic Press, 1966.
4.
L.A.Lustenik and V.J.Sobolev, Elements of Functional Analysis, Hindustan Publishing Corporation, New Delhi, 1971.
5.
J.B.Conway: A Course in Functional Analysis, Springer-Verlag, 1990.
6.
P.K.Jain, O.P.Ahuja and Khalil Ahmad: Functional Analysis, New Age International(P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.

SEMESTER- III
MM-502 Analytical Mechanics and Calculus of Variations
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.
SECTION-I (Two Questions)
Motivating problems of calculus of variations: shortest distance, Minimum surface of revolution,

Brachistochrone problem, Isoperimetric problem, Geodesic. Fundamental Lemma of calculus of
variation. Euler's equation for one dependent function of one and several independent variables,
and its generalization to (i) Functional depending on 'n' dependent functions, (ii) Functional
depending on higher order derivatives. Variational derivative, invariance of Euler's equations,
natural boundary conditions and transition conditions, Conditional extremum under geometric
constraints and under integral constraints . Variable end points.

SECTION-II (Two Questions)

Free and constrained systems, constraints and their classification. Generalized coordinates.
Holonomic and Non-Holonomic systems. Scleronomic and Rheonomic systems. Generalized
Potential, Possible and virtual displacements,ideal constraints. . Lagrange's equations of first
kind, Principle of virtual displacements D'Alembert's principle, HolonomicSystems independent
coordinates, generalized forces, Lagrange's equations of second kind. Uniqueness of solution.
Theorem on variation of total Energy. Potential, Gyroscopic and dissipative forces, Lagrange's
equations for potential forces equation for conservative fields.

SECTION-III (Two Questions)

Hamilton's variables. Don kin's theorem. Hamilton canonical equations. . Routh's equations.
Cyclic coordinates Poisson's Bracket. Poisson's Identity. Jacobi-Poisson theorem. Hamilton's
Principle, second form of Hamilton's principle. Poincare-Carton integral invariant. Whittaker's
equations. Jacobi's equations. Principle of least action

SECTION-IV (Two Questions)

Canonical transformations, free canonical transformations, Hamilton-Jacobi equation. Jacobi
theorem. Method of separation of variables for solving Hamilton-Jacobi equation. Testing the
Canonical character of a transformation. Lagrange brackets. Condition of canonical character of a
transformation in terms of Lagrange brackets and Poisson brackets. Simplicial nature of the Jacobian matrix of a canonical transformations. Invariance of Lagrange brackets and Poisson
brackets under canonical transformations.

Books:

1.
F. Gantmacher, Lectures in Analytic Mechanics, Khosla Publishing House, New
Delhi.

2.
H. Goldstein, Classical Mechanics (2nd edition), Narosa Publishing House, New
Delhi.
3. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
4. Francis B. Hilderbrand, Methods of applied mathematics , Prentice Hall,
5.

Narayan Chandra Rana & Pramod Sharad Chandra Joag. Classical Mechanics, Tata McGraw Hill, 1991.
6.
Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University
Press, 1998.

SEMESTER-III

MM-503 (opt. i) Elasticity

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Tensor Algebra: Coordinate-transformation, Cartesian Tensor of different order.
Properties of tensors, Isotropic tensors of different orders and relation between them,
Symmetric and skew symmetric tensors. Tensor invariants, Deviatoric tensors, Eigenvalues
and eigen-vectors of a tensor.
Tensor Analysis: Scalar, vector, tensor functions, Comma notation, Gradient, divergence
and curl of a vector / tensor field. (Relevant portions of Chapters 2 and 3 of book by D.S. Chandrasekharaiah and L Debnath)

SECTION-II (Two Questions)

Analysis of Strain : Affine transformation, Infinitesimal affine deformation, Geometrical Interpretation of the components of strain. Strain quadric of Cauchy. Principal strains and
invariance, General infinitesimal deformation. Saint-Venant's equations of compatibility.
Finite deformations
Analysis of Stress : Stress Vecotr, Stress tensor, Equations of equilibrium,
Transformation of coordinates.
(Relevant portion of Chapter I & II of book by I.S. Sokolnikoff).

SECTION-III (Two Questions)

Stress quadric of Cauchy, Principal stress and invariants. Maximum normal and shear stresses. Mohr's circles, examples of stress.
Equations of Elasticity : Generalised Hooks Law, Anisotropic symmetries, Homogeneous isotropic medium.
(Relevant portion of Chapter II & III of book by I.S. Sokolnikoff).

SECTION-IV (Two Questions)

Elasticity moduli for Isotropic media. Equilibrium and dynamic equations for an isotropic
elastic solid. Strain energy function and its connection with Hooke's Law, Uniqueness of solution. Beltrami-Michell compatibility equations. Clapeyrom's theorem. Saint-Venant's
principle.
(Relevant portion of Chapter III of book by I.S.Sokolnikoff).

Books:

1.

I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata-McGraw Hill Publishing Company Ltd., New Delhi, 1977.
2.
A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
3.
Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
4.
D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.
5.
Shanti Narayan, Text Book of Cartesian Tensor, S. Chand & Co., 1950.
6.
S. Timeshenki and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
7.
I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delhi, 1975.

SEMESTER-III
MM-503 (opt. ii) Difference Equations-I
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Introduction,the difference calculus: The difference operator,falling factorial power tr

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, summation, definition, properties and examples, Abel's
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summation formula, Generating functions, Euler's summation formula, Bernoulli polynomials and examples, approximate summation.

SECTION-II (Two Questions)

Linear Difference Equation: First order linear equations, general results for linear equations, solution of linear difference equation with constant coefficients and with variable coefficients, Non-Linear Equations that can be linearized, applications.

SECTION-III (Two Questions)

Stability Theory : Initial value Problems for Linear systems, eigen values, eigen vectors and spectral radius, Caylay-Hamilton Theorem, Putzer algorithm. Solution of nonhomogeneous system with initial conditions, Stability of linear systems, stable subspace theorem and example. Stability of non-linear system, Chaotic behaviour.

SECTION-IV (Two Questions)

The Z-Transform, definition, Properties, initial and final value Theorem, Convolation

Theorem, Solving the initial value problems, Volterra summation equation and Fredholm summation equation by use of Z-Transform.
Asymptotic Methods : Introduction, Asymptotic Analysis of Sums, and examples. Asymptotic behaviour of solutions of homogeneous linear equations, Poincare's Theorem, Perron Theorem (Statement only), non-linear equations.

Recommended Text:

W.G. Kelley and A.C. Peterson: Difference Equations; An introduction with Applications, Academic Press, Harcourt, 1991. (Relevant portions of chapters 1-5.)
Reference Book:
Calvin Ahlbrandt & Allan C. Peterson, Discreet Hamiltonian systems, Difference Equations, Continued Fractions & Ricati Equation, Kluwer Botson, 1996

SEMESTER-III
MM-503 (opt.iii) Analytic Number Theory
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.
SECTION-I (Two Questions)
Arithmetical functions, Mobius function, Euler totient function, relation connecting Mobius function and Euler totient function, Product formula for Euler totient function, Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, Mangoldt function, multiplicative functions, Multiplicative functions and Dirichlet multiplication. Inverse of completely multiplicative function, Liouville's function, divisor function, generalized convolutions, Formal power-series, Bell series of an arithmetical function, Bell series and Dirichlet multiplication, Derivatives of arithmetical functions, Selberg identity. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas, average order of divisor functions, average order of Euler totient function.
SECTION-II (Two Questions)
Application to the distribution of lattice points visible from the origin, average order of Mobius function and Mangoldt function, Partial sums of a Dirichlet Product, applications to Mobius function and Mangoldt function, Legendre's identity, another identity for the partial sums of a Dirichlet product. Chebyshev's functions, Abel's identity, some

equivalent forms of the prime number theorem. Inequalities for p(n) and P n .

SECTION-III (Two Questions)

Shapiro's Tauberian theorem. Applications of Shapiro's theorem. An asymptotic
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formula for the partial sums å çç ÷÷ . Partial sums of the Mobius function. Brief sketch

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of an elementary proof of the prime number theorem; Selberg's asymptotic formula.

Elementary properties of groups, construction of subgroups, characters of finite abelian groups, the character group, orthogonality relations for characters, Dirichlet characters, Sums-involving Dirichlet characters, Nonvanishing of L(1,c) for real nonprincipal c.

SECTION-IV (Two Questions)

Dirichlet's theorem for primes of the form 4n-1 and 4n+1. Dirichlet's theorem. Functions periodic modulo K, Existence of finite Fourier series for periodic arithmetical functions. Ramanujan's sum and generalizations, multiplicative properties of the sums S
k (n). Gauss sums associated with Dirichlet characters. Dirichlet characters with nonvanishing Gauss sums. Induced moduli and primitive characters, properties of induced moduli conductor of a character. Primitive characters and separable Gauss sums.

Finite fourier series of the Dirichlet characters. Polya's inequality for the partial sums of primitive characters.

Recommended Book:

Tom M. Apostol Introduction to Analytic Number Theory

SEMESTER-III

MM-503 (opt. iv) Number Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

The equation ax+by = c, simultaneous linear equations, Pythagorean triangles, assorted examples, ternary quadratic forms, rational points on curves.

SECTION-II (Two Questions)

Elliptic curves, Factorization using elliptic curves, curves of genus greater than 1. Farey sequences, rational approximations, Hurwitz theorem, irrational numbers, Geometry of Numbers, Blichfeldt's principle, Minkowski's Convex body theorem Lagrange's four square theorem.

SECTION-III (Two Questions)

Euclidean algorithm, infinite continued fractions, irrational numbers, approximations to irrational numbers, Best possible approximations, Periodic continued fractions, Pell's equation.

SECTION-IV (Two Questions)

Partitions, Ferrers Graphs, Formal power series, generating functions and Euler's identity, Euler's formula, bounds on P(n), Jacobi's formula, a divisibility property.

Recommended Text:

An Introduction to the Theory of Numbers
Ivan Niven
Herbert S. Zuckerman
Hugh L: Montgomery
John Wiley & Sons(Asia)Pte.Ltd.
(Fifth Edition)

SEMESTER- III
MM-504 (opt. i) Fluid Mechanics-I
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Kinematics of fluid in motion:Velocity at a point of a fluid. Lagrangian and Eulerian methods. Stream lines, path lines and streak liens, vorticity and circulation, Vortex lines, Acceleration and Material derivative, Equation of continuity (vector or Cartesian form).

Reynolds transport Theorem. General analysis of fluid motion. Properties of fluids-static and dynamic pressure. Boundary surfaces and boundary surface conditions. Inotational and rotational motions. Velocity potential.

SECTION-II (Two Questions)

Equation of Motion : Lagrange's and Euler's equations of Motion (vector or in Cartesian form). Bernculli's theorem. Applications of the Bernoulli Equation in one –dimensional flow problems. Kelvins circulation theorem, vorticity equation. Energy equation for incompressible flow. Kinetic energy of irrotational flow. Kelvins minimum energy theorem ,mean potential over a spherical surface. Kinetic energy of infinite liquid. Uniqueness theorems.

SECTION –III (Two Questions)

Stress components in a real fluid. Relations between rectangular components of stress. Connection between stresses and gradients of velocity.Navier-Stoke's equations of motion. Steady flows between two parallel plates, Plane Poiseuille and Couette flows.

SECTION –IV (Two Questions)

Reduction of Navier-Stock equations in flows having axis of symmetry, steady flow in circular pipe: the Hagen-Poiseuille flow, steady flow between two coaxial cylinders, flow between two concentric rotating cylinders. Steady flows through tubes of uniform cross-section in the form (i) Ellipse, (ii) equilateral triangle, (iii) rectangle, under constant pressure gradient, uniqueness theorem.

Books :

1.
W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers, Delhi, 1988.

2.
F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.

3.
Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, 1986.

4.
G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New Delhi, 1994.

5.

A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics
Springer-Verlag, New York, 1993.
6.
L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
7. H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New
York, 1979.
8.
R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing
Company, New Delhi. 1976.9
9.. A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.

10.
S. w. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New
Delhi, 1976.

Semester-III

MM : 504 (opt. ii) Mathematical Statistics
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Random distribution: preliminaries, Probability density function, Probability models, Mathematical Expectation, Chebyshev's Inequality; Conditional probability, Marginal and conditional distributions, Correlation coefficient, Stochastic independence.

Section-II (Two Questions)

Frequency distributions: Binomial, Poissson, Gamma, Chi-square, Normal, Bivariate normal distributions.
Distributions of functions: Sampling, Transformations of variables: discrete and continuous; t & F distributions; Change of variable technique; Distribution of order; Moment-generating function technique; other distributions and expectations.

Section-III (Two Questions)

Limiting distributions: Stochastic convergence, Moment generating function, Related theorems.
Intervals: Random intervals, Confidence intervals for mean, differences of means and variance; Bayesian estimation.

Section-IV (Two Questions)

Estimation & sufficiency: Point estimation, sufficient statistics, Rao-Blackwell Theorem, Completeness, Uniqueness, Exponential PDF, Functions of parameters; Stochastic independence.

Books:

1. R.V. Hogg & A.T. Craig:
Introduction to Mathematical Statistics, Amerind Pub.
Co. Pvt. Ltd. New Delhi, 1972. (Chapters 1 to 7)
2.

SC Gupta, VK Kapoor: Fundamentals of Mathematical Statistics, Sultan Chand & Sons (2007)

Semester – III

MM- 504 (opt. iii) Algebraic Coding Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION – I (Two Questions)

Block Codes. Minimum distance of a code. Decoding principle of maximum likelihood. Binary error detecting and error correcting codes. Group codes. Minimum distance of a group code (m, m+1) parity check code. Double and triple repition codes. Matrix codes.
Generator and parity check matrices. Dual codes. Polynomial codes. Exponent of a polynomial over the binary field. Binary representation of a number. Hamming codes. Minimum distance of a Hamming code. (Chapter 1, 2, 3 of the book given at Sr. No. 1).

SECTION – II (Two Questions)

Finite fields. Construction of finite fields. Primitive element of a finite field.
Irreducibility of polynomials over finite fields. Irreducible polynomials over finite fields.
Primitive polynomials over finite fields. Automorphism group of GF(qn). Normal basis of GF(qn). The number of irreducible polynomials over a finite field. The order of an irreducible polynomial. Generator polynomial of a Bose-Chaudhuri-Hocqhenghem codes (BCH codes) construction of BCH codes over finite fields. (Chapter 4 of the book given at Sr. No. 1 and Section 7.1 to 7.3 of the book given at Sr. No. 2).

SECTION – III (Two Questions)

Linear codes. Generator matrices of linear codes. Equivalent codes and permutation matrices. Relation between generator and parity-check matrix of a linear codes over a finite field. Dual code of a linear code. Self dual codes. Weight distribution of a linear code. Weight enumerator of a linear code. Hadamard transform. Macwilliams identity for binary linear codes.
Maximum distance separable codes. (MDS codes). Examples of MDS codes.
Characterization of MDS codes in terms of generator and parity check matrices. Dual code of a MDS code. Trivial MDS codes. Weight distribution of a MDS code. Number of code words of minimum distance d in a MDS code. Reed solomon codes. (Chapter 5 & 9 of the book at Sr. No. 1).

SECTION – IV (Two Questions)

Hadamard matrices. Existence of a Hadamard matrix of order n. Hadamard codes from Hadamard matrices Cyclic codes. Generator polynomial of a cyclic code. Check polynomial of a cyclic code. Equivalent code and dual code of a cyclic code. Idempotent generator of a cyclic code. Hamming and BCH codes as cyclic codes. Perfect codes. The Gilbert-varsha-move and Plotkin bounds. Self dual binary cyclic codes. (Chapter 6 & 11 of the book given at Sr. No. 1).

Recommended Text :

1. L.R. Vermani : Elements of Algebraic Coding Theory (Chapman and Hall Mathematics)
2. Steven Roman : Coding and Information Theory (Springer Verlag)

SEMESTER-III

MM-504 (opt. iv) Commutative Algebra

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Zero divisors, nilpotent elements and units, Prime ideals and maximal ideals, Nil radical and Jacobson radical, Comaximal ideals, Chineese remainder theorem, Ideal quotients and annihilator ideals. Extension and contraction of ideals. Exact sequences. Tensor product of module Restriction and extension of scalars. Exactness property of the tensor product. Tensor products of algebras.

SECTION-II (Two Questions)

Rings and modules of sections. Localization at the prime ideal P. Properties of the localization. Extended and contracted ideals in rings of fractions.
Primary ideals, Primary decomposition of an ideal, Isolated prime ideals, Multiplicatively closed subsets.

SECTION-III (Two Questions)

Integral elements, Integral closure and integrally closed domains, Going-up theorem and the Going-down theorem, valuation rings and local rings, Noether's normalization lemma and weak form of nullstellensatz Chain condition, Noetherian and Artinian modules, composition series and chain conditions.

SECTION-IV (Two Questions)

Noetherian rings and primary decomposition in Noetherian rings, radical of an ideal. Nil radical of an Artinian ring, Structure Theorem for Artinian rings, Discrete valuation rings, Dedekind domains, Fractional ideals.
(Scope of the course is as given in Chapter 1 to 9 of the recommended text).

Recommended Text:

M.F.Atiyah, FRS and I.G.Macdonald Introduction to Commutative Algebra (Addison-Wesley Publishing Company)

Reference Books:

1. N.S.Gopal Krishnan, Oxonian Press Pvt. Ltd. Commutative Algebra
2. Zariski, Van Nostrand Princeton(1958) Commutative Algebra(Vol. I)

SEMESTER-III
MM-505 (opt. i) Integral Equations
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Definition of Integral Equations and their classifications. Eigen values and Eigen functions. Special kinds of Kernel Convolution Integral. The inner or scalar product of two functions. Reduction to a system of algebraic equations. Fredholm alternative, Fredholm theorem, Fredholm alternative theorem, An approximate method.
(Relevant portions from the Chapters 1 & 2 of the book "Linear Integral Equations, Theory & Techniques by R.P.Kanwal").

SECTION-II (Two Questions)

Method of successive approximations, Iterative scheme for Fredholm and Volterrra Integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution. Some results about the resolvent Kernel. Application of iterative scheme to Volterra integral equations of the second kind.
Classical Fredholm's theory, the method of solution of Fredholm equation, Fredholm's First theorem, Fredholm's second theorem, Fredhom's third theorem.
(Relevant portions from the Chapter 3 & 4 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-III (Two Questions)

Symmetric Kernels, Introduction, Complex Hilbert space. An orthonormal system of functions, Riesz-Fisher theorem, A complete two-Dimensional orthonormal set over the rectangle a £ s £ b,c £ t £ d. Fundamental properties of Eigenvalues and Eigenfunctions for symmetric Kernels. Expansion in eigen functions and Bilinear form. Hilbert-Schmidt theorem and some immediate consequences. Definite Kernels and Mercer's theorem. Solution of a symmetric Integral Equation. Approximation of a general .
2 -Kernel(Not necessarily symmetric) by a separable

Kernel. The operator method in the theory of integral equations. Rayleigh-Ritz method for finding the first eigenvalue.
(Relevant portions from the Chapter 7 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-IV (Two Questions)

The Abel Intergral Equation. Inversion formula for singular integral equation with Kernel of the type h(s)-h(t), 0<a<1, Cauchy's principal value for integrals solution of the Cauchy-type singular integral equation, closed contour, unclosed contours and the Riemann-Hilbert problem. The Hilbert-Kernel, solution of the Hilbert-Type singular Intergal equation.

(Relevant portions from the Chapter 8 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

References:

1.
R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.

2.
S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book Agency, 1960.

3.
I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North Holland, 1966.

4.
I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II, Mac.Millan, 1969.

5.
Pundir and Pundir, Integral Equations and Boundary value problems, Pragati Prakashan, Meerut.

Semester-III

MM 505 : (opt. ii) Mathematical Modeling

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

The process of Applied Mathematics; mathematical modeling: need, techniques, classification and illustrative; mathematical modeling through ordinary differential equation of first order; qualitative solutions through sketching.

Section-II (Two Questions)

Mathematical modeling in population dynamics, epidemic spreading and compartment models; mathematical mode1ing through systems of ordinary differential equations; mathematical mode1ing in economics, medicine, arm-race, battle.

Section-III (Two Questions)

Mathematical modeling through ordinary differential equations of second order. Higher order (linear) models. Mathematical modeling through difference equations: Need, basic theory; mathematical modeling in probability theory, economics, finance, population dynamics and genetics.

Section-IV(Two Questions)

Mathematical modeling through partial differential equations: simple models, mass-balance equations, variational principles, probability generating function, traffic flow problems, initial & boundary conditions.

Book recommended :

J.N. Kapur: Mathematical Modeling, Wiley Eastern Limited, 1990 (Relevant portions, mainly from Chapters 1 to 6.)

Semester – III

MM-505 (opt. iii) LINEAR PROGRAMMING

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Questions)

Simultaneous linear equations, Basic solutions, Linear transformations, Point sets, Lines and hyperplanes, Convex sets, Convex sets and hyperplanes, Convex cones, Restatement of the Linear Programming problem, Slack and surplus variables, Preliminary remarks on the theory of the simplex method, Reduction of any feasible solution to a basic feasible solution, Definitions and notations regarding linear programming problems. Improving a basic feasible solution, Unbounded solutions, Optimality conditions, Alternative optima, Extreme points and basic feasible solutions.

Section-II (Two Questions)

The simplex method, Selection of the vector to enter the basis, Degeneracy and breaking ties, Further development of the transformation formulas, The initial basic feasible solution-----artificial variables, Inconsistency and redundancy, Tableau format for simplex computations, Use of the tableau format, Conversion of a minimization problem to a maximization problem, Review of the simplex method.
The two-phase method for artificial variables, Phase I, Phase II, Numerical examples of the two-phase method, Requirements space, Solutions space, Determination of all optimal solutions, Unrestricted variables, Charnes' perturbation method regarding the resolution of the degeneracy problem.

Section-III (Two Questions)

Selection of the vector to be removed, Definition of b(€). Order of vectors in b(€), Use of perturbation technique with simplex tableau format, Geometrical interpretation of the perturbation method. The generalized linear programming problem, The generalized simplex method, Examples pertaining to degeneracy, An example of cycling.
Revised simplex method: Standard Form I, Computational procedure for Standard Form I, Revised simplex method: Standard Form II, Computational procedure for Standard Form II, Initial identity matrix for Phase I, Comparison of the simplex and revised simplex methods, The product form of the inverse of a non-singular matrix. Alternative

formulations of linear programming problems,

Section-IV (Two Questions)

Dual linear programming problems, Fundamental properties of dual problems, Other formulations of dual problems,Complementary slackness, Unbounded solution in the primal, Dual simplex algorithm, Alternative derivation of the dual simplex algorithm, Initial solution for dual simplex algorithm, The dual simplex algorithm; an example, geometric interpretations of the dual linear programming problem and the dual simplex algorithm. A primal dual algorithm, Examples of the primal-dual algorithm. Transportation problem, its formulation and simple examples.

Books :

1. G.Hadley : Linear Programming Narosa publishing House (1995)
2. S.I. Gauss : Linear Programming : Methods and Applications (4th Edition) McGraw Hill, New York 1975

SEMESTER-III

MM 505 (opt. iv) Fuzzy Sets and Applications-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Fuzzy Sets: Basic definitions, a-cuts, strong a-cuts, level set of a fuzzy set, support of a fuzzy set, the core and height of a fuzzy set, normal and subnormal fuzzy sets, convex fuzzy sets, cutworthy property, strong cutworthy property, standard fuzzy set operations, standard complement, equilibrium points, standard intersection, standard union, fuzzy set inclusion, scalar cardinality of a fuzzy set, the degree of subsethood (Scope as in relevant parts of sections 1.3-1.4 of Chapter 1 of the book given at Sr.No.1).

Additional properties of a-cuts involving the standard fuzzy set operators and the standard fuzzy set inclusion, Representation of fuzzy sets, three basic decomposition theorems of fuzzy sets Extension principle for fuzzy sets: the Zedah's extension principle, Images and inverse images of fuzzy sets, proof of the fact that the extension principle is strong cutworthy but not cutworthy (Scope as in relevant parts of Chapter 2 of the book mentioned at the end).

SECTION-II (Two Questions)

Operators on fuzzy sets: types of operations, fuzzy complements, equilibrium of a fuzzy complement, equilibrium of a continuous fuzzy complement, first and second characterization theorems of fuzzy complements, fuzzy intersections (t-norms), standard

fuzzy intersection as the only idempotent t-norm, standard intersection, algebraic product,
bounded difference and drastic intersection as examples of t-norms, decreasing generator,
the Pseudo-inverse of a decreasing generator, increasing generators and their Pseudo-inverses, convertion of decreasing generators and increasing generators to each other,
characterization theorem of t-norms(statement only). Fuzzy unions(t-conorms), standard
union, algebraic sum, bounded sum and drastic union as examples of t-conorms,
characterization theorem of t-conorms (Statement only) (Scope as in relevant parts of
sections 3.1 to 3.4 of Chapter 3 of the book mentioned at the end).

SECTION-III (Two Questions)

Fuzzy numbers, relation between fuzzy number and a convex fuzzy set, characterization
of fuzzy numbers in terms of its membership functions as piecewise defined functions,
fuzzy cardinality of a fuzzy set using fuzzy numbers, arithmetic operators on fuzzy
numbers, extension of standard arithmetic operations on real numbers to fuzzy numbers.
lattice of fuzzy numbers, (R, MIN, MAX) as a distributive lattice, fuzzy equations,
equation A+X = B, equation A.X = B (Scope as in relevant parts of sections Chapter 4 of book mentioned at the end).

SECTION-IV (Two Questions)

Fuzzy Relations: Crisp and fuzzy relations, projections and cylindrical extensions, binary fuzzy relations, domain, range and height of a fuzzy relation, membership matrices,
sagittal diagram, inverse of a fuzzy relation, composition of fuzzy relations, standard composition, max-min composition, relational join, binary relations on a single set,
directed graphs, reflexive irreflexive, antireflexive, symmetric, asymmetric,
antisymmetric, transitive (max-min transtitive), non transitive, antitransitive fuzzy
relations.
Fuzzy equivalence relations, fuzzy compatibility relations, a-compatibility class, maximal a-compatibles, complete a-cover, reflexive undirected graphs, fuzzy ordering
relations, fuzzy upper bound, fuzzy pre ordering, fuzzy weak ordering, fuzzy strict
ordering, fuzzy morphisms.

(Scope of this section is as in the relevant parts of sections 5.1 to 5.8 of Chapter 5 of the book mentioned at the end).

Recommended Text:

G.J.Klir and B.Yuan: Fuzzy Sets and Fuzzy Logic; Theory and Applications, Sixth Indian Reprint, Prentice Hall of India, New Delhi, 2002.

Semester-III

Paper MM- 506 : Practical-III

Examination Hours : 4
hours
Max. Marks : 100
Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-501 to MM-505
will be taught.
Part-B : Implementation of the following programs in FORTRAN-90/95

1.

Use a function program for simple interest to display year-wise compound interest and amount, for given deposit, rate and time.

2.

Use logical operators in computing the compound interest on a given amount for rate of interest varying with amount as well as time of deposit.

3.

Write a subroutine program to check (logical output) whether the three given points in a plane are collinear.

4.

Use subroutine program to multiply two given matrices and use resource files in main program to read input and write output.

5.

Use ALLOCATABLE size declaration for given set of points in a plane and fit a straight line through these points.

6.

Write a program to display the use of whole-array operations on non-conformable arrays.

7.

Write a program to display the procedure of format-rescan-rule and the action of tab-edit descriptors.

8.

Use string operations to find if a given string is a palindrome or not.

9.

Compute a given definite integral (as summation) in a subroutine using integrand as a dummy argument.

10. Explain the use of MODULE in defining an abstract (derived) data type for complex arithmetic.

11. Use of pointers in manipulating a linked-list.

12. To solve a quadratic equation with given (complex-valued) coefficients, using COMPLEX data type

Note :-Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee’s performance in written solutions/programs, execution of computer programs and viva-voce examination.

SEMESTER-IV

MM-507 General Measure and Integration Theory

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Measures, some properties of measures, outer measures, extension of measures, uniqueness of extension, completion of a measure, the LUB of an increasingly directed family of measures.(Scope as in the Sections 3-6, 9-10 of Chapter 1 of the book 'Measure and Integration' by S.K.Berberian).

Measurable functions, combinations of measurable functions, limits of measurable functions, localization of measurability, simple functions (Scope as in Chapter 2 of the book 'Measure and Integration' by S.K.Berberian).

SECTION-II (Two Questions)

Measure spaces, almost everywhere convergence, fundamental almost everywhere, convergence in measure, fundamental in measure, almost uniform convergence, Egoroff's theorem, Riesz-Weyl theorem (Scope as in Chapter 3 of the book 'Measure and Integration' by S.K.Berberian).

Integration with respect to a measure: Integrable simple functions, non-negative integrable functions, integrable functions, indefinite integrals, the monotone convergence theorem, mean convergence (Scope as in Chapter 4 of the book 'Measure and Integration' by S.K.Berberian)

SECTION-III (Two Questions)

Product Measures: Rectangles, Cartesian product of two measurable spaces, measurable rectangle, sections, the product of two finite measure spaces, the product of any two measure spaces, product of two s -finite measure spaces; iterated integrals, Fubini's theorem, a partial converse to the Fubini's theorem (Scope as in Chapter 6 (except section 42) of the book 'Measure and Integration' by S.K.Berberian)

Signed Measures: Absolute continuity, finite singed measure, contractions of a finite signed measure, purely positive and purely negative sets, comparison of finite measures, Lebesgue decomposition theorem, a preliminary Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, upper variation, lower variation, total variation, domination of finite signed measures, the Radyon-Nikodym theorem for a finite measure space, the Radon-Nikodym theorem for a s -finite measure space (Scope as in Chapter 7 (except Section 53) of the book 'Measure and Integration' by S.K.Berberian).

SECTION-IV (Two Questions)

Integration over locally compact spaces: continuous functions with compact support, Gd 's and Fs's, Baire sets, Baire function, Baire-sandwich theorem, Baire measure, Borel sets, Regularity of Baire measures, Regular Borel measures, Integration of continuous functions with compact support, Riesz-Markoff's theorem (Scope as in relevant parts of the sections 54-57,60,62,66 and 69 of Chapter 8 of the book 'Measure and Integration' by S.K.Berberian)

Recommended Text:

S.K.Berberian: Measure and Integration, Chelsea Publishing Company, New York, 1965.

References:

1.

H.L.Royden: Real Analysis, Prentice Hall of India, 3rd Edition, 1988.
2.
G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd.,1981.
3.
P.R.Halmos: Measure Theory, Van Nostrand, Princeton, 1950.
4.
I.K.Rana: An Introduction to Measure and Integration, Narosa Publishing
House, Delhi, 1997.
5.
R.G.Bartle: The Elements of Integration, John Wiley and Sons, Inc. New
York, 1966.

SEMESTER- IV

MM-508 Partial Differential Equations

Examination
Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The
candidate is required to attempt five questions selecting at least one from each
section and the compulsory question.
SECTION-I (Two Questions)
PDE of kth order: Definition, examples and classifications. Initial value problems. Transport
equations homogeneous and non-homogeneous, Radial solution of Laplace's Equation:
Fundamental solutions, harmonic functions and their properties, Mean value Formulas, Poisson,s
equation and its solution, strong maximum principle, uniqueness, local estimates for harmonic
functions, Liouville,s theorem, Harnack's inequality.

SECTION-II (Two Questions)

Green's function and its derivation, representation formula using Green's function, symmetry of
Green's function, Green's function for a half space and for a ball. Energy methods: uniqueness,
Drichlet's principle. Heat Equations: Physical interpretation, fundamental solution. Integral of
fundamental solution , solution of initial value problem, Duhamel's principle, non-homogeneous
heat equation, Mean value formula for heat equation, strong maximum principle and uniqueness.
Energy methods.

SECTION-III (Two Questions)
Wave equation-Physical interpretation, solution for one dimentional wave equation, d'Alemberts
formula and its applications, reflection method, Solution by spherical means Euler-
Poisson_Darboux equation, Kirchhoff's and Poisson's formulas (for n=2, 3 only), Solution of non
–homogeneous wave equation for n=1,3. Energy method. Uniqueness of solution, finite

propagation speed of wave equation. Non-linear first order PDE-complete integrals, envelopes,
Characteristics of (i) linear, (ii) quasilinera, (iii) fully non-linear first order partial differential
equations. Hamilton Jacobi equations (calculus of variations Hamilton's ODE, Legendre
Transform, Hopf-Lax formula, weak solutions, Uniqueness).

SECTION-IV (Two Questions)

Conservative Laws (Shocks, entropy condition, Lax-Oleinik formula., weak solutions uniqueness.
Riemann's problem, long time behaviuor).
Representation of Solutions-Separation of variables, Similarity solutions (Plane and traveling
waves, solitones, similarity under Scaling). Fourier Transform, Laplace Transform, Converting
non linear into linear PDE, Cole-Hop Transform, Potential functions, Hodograph and Legendre
transforms.

Books:

1L.C. Evans, Partial Differential Equations, Graduate Studies in
2 Books with the above title by I.N. Snedden, F. John, P. Prasad and R. Ravindran, Amarnath etc.

SEMESTER-IV
MM-509 (opt. i) Mechanics of Solids
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The
candidate is required to attempt five questions selecting at least one from each
section and the compulsory question.
SECTION-I (Two Questions)
Two dimensional problems : Plane stress. Generalized plane stress. Airy stress function. General solution of biharmonic equation, Stresses and displacements in terms of complex
potentials. The structure of functions of f(z) and .(z). First and second boundary-value
problems in plane elasticity. Existence and uniqueness of the solutions.
(Section 65-74 of the book by I.S. Sokolnikoff).

SECTION -II (Two Questions)

Waves : Propagation of waves in an isotropic elastic solid medium. Waves of dilatation
and distortion. Plane waves. Elastic surface waves : Rayleigh waves and Love waves.
Extension : Extension of beams, bending of beams by own weight and terminal couples,;
bending of rectangular beams
(Section 204 of A.E.H. Love, Sections 7,7-8, 10 of Y.C. Fung; Chapter 4, Sections 30 to
32 and 57 of book by I.S. Sokolnikoff).

SECTION -III (Two Questions)

Torsion : Torsion of cylindrical bars; Torsional rigidity. Torsion and stress functions.

Lines of shearing stress. Torsion of anisotropic beams; Simple problems related to circle,
ellipse and equilateral triangle.
(Chapter 4: Sections 33 to 38 and 51 of the book; I.S. Sokolnikoff, Section 221 of

A.E.H. Love).

SECTION -IV(Two Questions)

Variational methods : Theorems of minimum potential energy. Theorems of minimum complementary energy. Reciprocal theorem of Betti and Rayleigh. Deflection of elastic string cental line of a beam and elastic membrane. Solution of Euler's equation by Ritz,
Galerkin and Kantorovich methods.
(Chapter 7: Sections 107-110, 112, 113, 115 & 117 of I.S. Sokolnikoff).

Books:

1.
I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw Hill Publishing Company Ltd., New Delhi, 1977.
2.
A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
3.
Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
4.
D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.
5.
S. Timeshenki and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
6.
I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delh, 1975.

SEMESTER-IV

MM-509 (opt. ii) Difference Equations-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

The self-adjoint second order linear equations: Introduction, Lagrange identity, Green’s Theorem, Liouville’s formula, Polya Factorization Theorem and application, Cauchy function, variation of constants formula.
Sturmian Theory : Sturm separation theorem and examples. The Riccati Equation.

SECTION-II (Two Questions)

Sturm comparison Theorem. Oscillation.
The Sturm-Liouville problem : Introduction, eigen functions and eigen values of Sturm-Liouville problem, Finite Fourier analysis, Non-homogeneous problem. Rayleigh’s inequality.

SECTION-III (Two Questions)

Green's functions and Boundary Value Problems, Disconjugacy. B.V.P. for non-linear equation : Inrtroduction, contraction mapping theorem. Lipschitz condition & examples. Existence of solutions, some basic theorem and examples. B.V.P. for Differential Equations.

SECTION-IV (Two Questions)

Discrete calculus of variation, Introduction, Necessary condition for the simplest variational problem of local extremum, Euler-Lagrange equation, Sufficient condition and Disconjugacy, Sturm comparison Theorem, Weiesstrass Summation formula. Partial Differential Equations, Discretization of Partial Differential Equations, Solution of Partial Differential Equation.

Recommend Text:

W.G. Kelley and A.C. Peterson: Difference Equations; An introduction with Applications, Academic Press, Harcourt, 1991. (Relevant portions of chapters 6-10.)

Reference Book:

Calvin Ahlbrandt & Allan C. Peterson, Discreet Hamiltonian systems, Difference Equations, Continued Fractions & Ricati Equation, Kluwer Botson, 1996.

SEMESTER-IV

MM-509 (opt. iii) Algebraic Number Theory

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Algebraic numbers and algebraic integers. Transcendental Numbers. Liouville's Theorem for real Algebraic numbers. Thue Theorem and Roth's theorem (statement only). Algebraic numberfield K. Theorem of Primitive elements. Liouville's Theorem for complex algebraic numbers. Minimal polynomial of an algebraic integer. Primitive m-th roots of unity. Cyclotomic Polynomials. Norm and trace of algebraic numbers and algebraic integers. Bilinear form on algebraic number field K.

SECTION-II (Two Questions)

Integral basis and discriminant of an algebraic number field. Index of an element of K. Ring Ok of algebraic integers of an algebraic number field K. Ideals in the ring of algebraic number field K. Integrally closed domains. Dedekind domains. Fractional ideals of K. Factorization of ideals as a product of prime ideals in the ring of algebraic integers of an algebraic number field K. G.C.D. and L.C.M. of ideals in Ok. Chinese Remainder theorem.

SECTION-III (Two Questions)

Different of an algebraic number field K. Dedekind theorem. Euclidean rings. Hurwitz Lemma and Hurwitz constant. Equivalent fractional ideals. Ideal class group. Finiteness of the ideal class group. Class number of the algebraic number field K. Diophantine equations Minkowski's bound.

SECTION-IV (Two Questions)
Quadratic reciprocity Legendre Symbol. Gauss sums. Law of quadratic reciprocity.
Quadratic fields. Primes in special progression.
Recommended Text:

Jody Esmonde and M.Ram Murty
Problems in Algebraic Number Theory
(Springer Verlag, 1998)

Reference Books:

1. Paulo Ribenboim
Algebraic Numbers
2. R. Narasimhan
Algebraic Number Theory
and S. Raghavan
Mathematical Pamphlets-4. Tata Institute of Fundamental Research(1966).

Semester-IV
MM-509 Option (iv): Mathematics for Finance & Insurance
Examination Hours : 3 Hours
Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.
Section – I (Two Questions)
Normal Random Variables : Continuous Random Variables, Normal Random Variables
& their properties, Central Limit Theorem.
Geometric Brownian Motion : Basic concepts & simple Models, Brownian Motion.
Interest Rates, Present Value Analysis, Rate of Return, continuously varying Interest Rates.

Section – II (Two Questions)

Financial Derivatives – An Introduction, Types of Financial Derivatives, Forwards and Futures, Options and its kinds and SWAPS
The Arbitrage Theorem and Introduction to Portfolio Selection and Capital Market
Theory: Static and Continuous-Time Model.

Section – III (Two Questions)

Pricing by Arbitrage-A Single-Period option Pricing Model; Multi-Period Pricing Model,
Cox-Ross-Rubinstein Model; Bounds on Option Prices.
The Ito's Lemma and the Ito's Integral.
Concepts from Insurance: Introduction; The Claim Number Process; The Claim Size Process; Solvability of the Portfolio;
Reinsurance and Ruin Problem.

Section – IV (Two Questions)

Premium and Ordering of Risks-Premium Calculation Principles and Ordering
Distributions.
Distribution of Aggregate Claim Amount-Individual and Collective Model; Compound
Distributions; Claim Number of Distributions; Recursive Computation Methods;
Lundberg Bounds and Approximation by Compound Distributions.

References:

1.
John C.Hull, Options, Futures, and Other Derivatives, Prentice-Hall of India
Private Limited.
2.
Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge
University Press.
3.
Salih N. Neftci, An Introduction to the Mathematics of Financial Derivatives,
Academic Press, Inc.
4.
Robert J. Elliott and P. Ekkehard Kopp. Mathematics of Financial Markets,
Springer-Verlag, New York Inc.
5.
Robert C. Marton, Continuous-Time Finance, Basil Blackwell Inc.
6.
Daykin C.D., Pentikainen T. and Pesonen M., Practical Risk Theory for
Actuaries, Chapman & Hall.

SEMESTER- IV

MM-510 (opt. i) Fluid Mechanics –II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory question
will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Fundamental Equations: Derivation of the equations of continuity and equation of motion
in cylindrical and spherical coordinates. Two–dimentional inviscid incompressible flows, Stream function : Irrotatonal motion in two dimensions. Complex velocity potential. Sources, sinks, doublets and their images. Thomson circle theorem. Two-dimensional irrotational motion produced by motion of circular cylinder.

SECTION-II (Two Questions)

Two dimensional motion : Motion due to elliptic cylinder in an infinite mass of liquid, Kinetic energy of liquid contained in rotating elliptic cylinder, circulation about elliptic cylinder. Theorem of Blasius. Theorem of Kutta and Joukowski. Kinetic energy of a cyclic and acyclic irrotational motion. Axisymmetric flows, Stoke's stream function ,Stoke's stream functions of some basic flows.

SECTION-III (Two Questions)

Three –dimential motion : Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion a sphere. Alembert's paradox, impulsive motion, initial motion of liquid contained in the intervening space between two
concentric spheres. Vortex motion and its elementary properties. Kelvin's proof of permanence. Motions due to circular and rectilinear vortices. Infinite rowes of line vortices.

SECTION-IV (Two Questions)

Dynamical similarity . Buckingham pi-theorem , Reynolds number. Prandtl's boundary layer, boundary layer equations in two dimensions. Blasius solution Boundary layer thickness. Displacement thickness, Karman integral conditions, separation of boundary layer.

Books :

1. W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers,
Delhi, 1988.

2.
F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
3.
Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics,
John Wiley & Sons, 1986.
4.
G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New
Delhi, 1994.
5.
A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics
Springer-Verlag, New York, 1993.
6.
L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
7.
H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1979.
8.
R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing
Company, New Delhi. 1976.
9.
A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.
10.
S. w. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New
Delhi, 1976.event

SEMESTER-IV
MM-510 (opt.ii) Boundary Value Problems
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Applications to Ordinary Differential Equations; Initial value problems, Boundary Value Problems. Dirac Delta functions. Green's function approach to reduce boundary value problems of a self-adjoint-differential equation with homogeneous boundary conditions to integral equation forms. Green's function for N th -order ordinary differential equation. Modified Green's function.
(Relevant portions from the Chapter 5 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-II (Two Questions)

Applications to partial differential equations: Integral representation formulas for the solution of the Laplace and Poisson Equations. The Newtonian, single-layer and double-

layer potentials, Interior and Exterior Dirichlet problems, Interior and Exterior Neumann
problems. Green's function for Laplace's equation in a free space as well as in a space
bounded by a ground vessel. Integral equation formulation of boundary value prolems
for Laplace's equation. Poisson's Integral formula. Green's function for the space
bounded by grounded two parallel plates or an infinite circular cylinder. The Helmholtz
equation.
(Relevant portions from the Chapter 6 of the book "Linear Integral Equation, Theory and
Techniques by R.P.Kanwal").

SECTION-III (Two Questions)

Integral Transform methods: Introduction, Fourier transform. Laplace transform.
Convolution Integral. Application to Volterra Integral Equations with convolution-type
Kernels. Hilbert transform.
Applications to mixed Boundary Value Problems: Two-part Boundary Value problems,
Three-part-Boundary Value Problems, Generalized Three-part Boundary Value
problems.
(Relevant portions from the Chapter 9 and 10 of the book "Linear Integral Equation,
Theory and Techniques by R.P.Kanwal").

SECTION-IV (Two Questions)

Integral equation perturbation methods: Basic procedure, Applications to Electrostatics,
Low-Reynolds-Number Hydrodynamics: Steady stokes Flow, Boundary effects on
Stokes flow, Longitudnal oscillations of solids in stokes Flow, Steady Rotary Stokes
Flow, Rotary Oscillations in Stokes Flow, Rotary Oscillation in Stokes Flow, Oseen
Flow-Translation Motion, Oseen Flow-Rotary motion Elasticity, Boundary effects,
Rotation, Torsion and Rotary Oscillation problems in elasticity, crack problems in
elasticity, Theory of Diffraction.
(Relevant portions from the Chapter 11 of the book "Linear Integral Equation, Theory
and Techniques by R.P.Kanwal").

References:

1.
R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press,
New York.
2.
S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book
Agency, 1960.
3.
I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North
Holland, 1966.
4.
I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II,
Mac.Millan, 1969.
5.
Pundir and Pundir, Integral equations and Boundary value problems, Pragati
Prakashan, Meerut.

SEMESTER-IV

MM-510 (opt. iii) Non-Commutative Rings

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Basic terminology and examples of non-commutative rings i.e. Hurwitz's ring of integral quaternions, Free k-rings. Rings with generators and relations. Hilbert's Twist, Differential polynomial rings, Group rings, Skew group rings, Triangular rings, D.C.C. and A.C.C. in triangular rings. Dedekind finite rings. Simple and semi-simple modules and rings. Spliting homomorphisms. Projective and Injective modules. (Section 1.1 to 1.26 and Section 2.1 to 2.9 of the book given at Sr. No. 1).

SECTION-II (Two Questions)

Ideals of matrix ring M n (R). Structure of semi simple rings. Wedderburn-Artin Theorem Schur's Lemma. Minimal ideals. Indecomposable ideals. Inner derivation d. d-simple rings. Amitsur Theorem on non-inner derivations. Jacobson radical of a ring R.Annihilator ideal of an R-module M. Jacobson semi-simple rings. Nil and Nilpotent ideals. Hopkins-Levitzki Theorem. Jacobson radical of the matrix ring M n (R). Amitsur Theorem on radicals. Nakayama's Lemma. Von Neumann regular rings. E. Snapper's Theorem. Amitsur Theorem on radicals of polynomial rings. (Section 3.1 to 3.19, Sections 4.1 to 4.27, Section 5.1 to 5.10 of the book given at Sr. No. 1).

SECTION-III (Two Questions)

Prime and semi-prime ideals. m-systems. Prime and semi-prime rings. Lower and upper nil radical of a ring R Amitsur theorem on nil radical of polynomial rings. Brauer's Lemma. Levitzki theorem on nil radicals. Primitive and semi-primitive rings. Left and right primitive ideals of a ring R. Density Theorem. Structure theorem for left primitive rings. (Section 10.1 to 10.30, Section 11.1 to 11.20 of the book given at Sr. No. 1).

SECTION-IV (Two Questions)

Sub-direct products of rings. Subdirectly reducible and irreducible rings. Birchoff's Theorem. Reduced rings. G.Shin's Theorem. Commutativity Theorems of Jacobson, Jacobson-Herstein and Herstein Kaplansky. Division rings. Wedderburn's Little Theorem. Herstein's Lemma. Jacobson and Frobenius Theorem. Cartan-Brauer-Hua Theorem. Herstein's Theorem. (Sections 12.1 to 12.11 and Sections 13.1 to 13.26 of the book given at Sr. No. 1).

Recommended Book:

1. T.Y.Lam
A First Course in Noncommutive Rings, (Springer Verlag 1990)
2.
I.N.Herstein Non-Commutative Rings carus monographs in Mathematics
Vol.15. Math Asso. of America 1968.

SEMESTER-IV
MM-510 (opt. iv) Advanced Discrete Mathematics
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Graphs, Konisberg seven bridges problem. Finite and infinite graphs. Incidence vertex. Degree of a vertex. Isolated and pendant vertices. Null graphs. Isomosphism of graphs. Subgraphs, walks, paths and circuits. Connected and disconnected graphs. Components of a graph. Euler graphs. Hamiltonian paths and circuits. The traveling salesman problem. Trees and their properties. Pendant vertices in a tree. Rooted and binary tree. Spanning tree and fundamental circuits. Spanning tree in a weighted graph. (Chapter 1,2,3 of the book given at Sr. No. 1).

SECTION-II (Two Questions)

Cutsets and their properties. Fundamental circuits and cutsets. Connectivity and separability. Network flows. Planner graphs. Kuratowski's two graphs. Representation of planner graphs. Euler formula for planner graphs. Vector space associated with a graph. Basis vectors of a graph. Circuit and cutset subspaces. Intersection and joins of WC and WS. Incidence matrix A(G) of a graph G, Submatrices of A(G), Circuit matrix, Fundamental circuit matrix, and its rank, Cutset matrix, path matrix and adjacency matrix of a graph. (Chapter 4, Theorems 5.1 to 5.6 of chapter 5, chapter 6 & 7 of the book given at Sr. No. 1).

SECTION-III (Two Questions)

Partially ordered sets and lattices. Lattice as an algebraic system. Sublattices. Isomorphism of lattices. Distributive and modular lattices. Lattices as intervals. Similar and projective intervals. Chains in lattices. Zassenhaus's Lemma and Schreier Theorem, Composition chain and Jordan Holder Theorem. Chain conditions. Fundamental dimensionality relation for modular lattices. Decomposition theory for lattices with ascending chain conditions, i.e. reducible and irreducible elements. Independent elements in lattices. (Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

SECTION-IV (Two Questions)

Points (atoms) of a lattice. Complemented lattices. Chain conditions and complemented lattices. Boolean algebras. Conversion of a Boolean algebra into a Boolean ring with unity and vice-versa. Direct product of Boolean algebras. Uniqueness of finite Boolean algebras. Boolean functions and Boolean expressions. Application of Boolean algebra to switching circuit theory. (Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

Recommended Texts:

1.
Narsingh Deo Graph Theory with application to Engineering and Computer Science, Prentice Hall of India.

2.
Nathan Jacobson Lectures in Abstract Algebra Vol.I, D.Van Nostrand Company, Inc.

3. L.R. Vermani and
A course in discrete Mathematical structures(Imperial College

Shalini Press London 2011)

SEMESTER-IV

MM-511 (opt. i) Mathematical Aspects of Seismology

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.
SECTION-I (Two Questions)

General form of progressive waves, Harmonic waves, Plane waves, the wave equation.
Principle of superposition. Special types of solutions: Progressive and Stationary type solutions of wave equation. Equation of telegraphy. Exponential form of harmonic waves. D' Alembert's formula. Inhomogeneous wave equation. Dispersion: Group velocity, relation between phase velocity and group velocity.
(Relevant articles from the book "Waves" by Coulson & Jefferey)

SECTION-II (Two Questions)

Reduction of equation of motion to wave equations. P and S waves and their characteristics. Polarisation of plane P and S waves. Snell's law of reflection and refraction. Reflection of plane P and SV waves at a free surface. Partition of reflected energy. Reflection at critical angles.
Reflection and reflection of plane P,SV and SH waves at an interface. Special cases of Liquid-Liquid interface, Liquid-Solid interface and Solid-Solid interface. Rayleigh waves, Love waves and Stoneley waves. (Relevant articles from the book, "Elastic waves in Layered Media" by Ewing et al).

SECTION-III (Two Questions)

Two dimensional Lamb's problems in an isotropic elastic solid: Area sources and Line Sources in an unlimited elastic solid. A normal force acts on the surface of a semi-infinite elastic solid, tangential forces acting on the surface of a semi-infinite elastic solid.
Three dimensional Lamb's problems in an isotropic elastic solid: Area sources and Point sources in an unlimited elastic solid, Area source and Point source on the surface of semi-infinite elastic solid.
Haskell matrix method for Love waves in multilayered medium.
(Relevant articles from the book "Mathematical Aspects of Seismology" by Markus Bath).

SECTION-IV (Two Questions)

Spherical waves. Expansion of a spherical wave into plane waves: Sommerfield's integral. Kirchoff's solution of the wave equation, Poissons's formula, Helmholtz's formula.

(Relevant articles from the book "Mathematical Aspects of Seismology" by Markus Bath).

Introduction to Seismology: Location of earthquakes, Aftershocks and Foreshocks, Earthquake magnitude, Seismic moment, Energy released by earthquakes, observation of earthquakes, interior of the earth.

(Relevant articles from the book "The Solid Earth" by C.M.R.Fowler)

References:

1. P.M.Shearer, Introduction to Seismology, Cambridge University Press,(UK) 1999.
2. C.M.R.Fowler, The Solid Earth, Cambridge University Press, 1990.
3. C.A.Coulson and A.Jefferey, Waves, Longman, New York, 1977.
4. M.Bath, Mathematical Aspects of Seismology, Elsevier Publishing Company, 1968.
5. W.M.Ewing, W.S.Jardetzky and F.Press, Elastic Waves in Layered Media, McGraw Hill Book Company, 1957.

SEMESTER-IV

MM-511 (opt. ii) Dynamical Systems

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to two questions from each section and one compulsory question consisting of eight parts and distributed over the whole syllabus. An examinee is required to attempt one question from each section and the compulsory question.

Section-I

Orbit of a map; fixed point; Periodic point; Circular map, Configuration space & phase space.

Section-II

Origin of bifurcation; Stability of a fixed point, equilibrium point; Concept of limit cycle & torus; Hyperbolicity; Quadratic map; Feigenbaum's universal constant.

Section-III

Turning point, transcritical, pitch work; Hopf bifurcation; Period doubling phenomenon. Non-linear oscillators

Section-IV

Conservative system; Hamiltonian system; Various types of oscillators; Solutions of nonlinear differential equations.

Books :

1. D.K. Arrowosmith, Introduction to Dynamical Systems, CUP, 1990.
2. R.L Davaney, An Introduction to Chaotic Dynamical Systems, Addison-Wesley, 1989.
3. P.G. Drazin, Nonlinear System, CUP, 1993.
4. V.I Arnold, Nonlinear Systems III- Mathematical Aspects of Classical and Celestial Mechanics, Springer-Verlag, 1992.
5. V.I Arnold, Nonlinear Systems V-Bifurcation Theory and Catastrophe Theory, Springer-Verlag, 1992.

Semester-IV

MM-511 (opt. iii) Operational Research
Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)
NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Question)

Dynamic Programming – Nature of Dynamic Programming (DP), Bellman's principle of optimality in DP, DP algorithm, mathematical formulation of multistage model, the recursive operation approach, Application of DP in Linear Programming.
Integer Programming : types of integer programming problem, cutting plane method (Gomory technique), construction of Gomory's constraints, Graphical interpretation of cutting plane method, cutting plane algorithm, Fractional cut methodthe branch and bound method.

Section – II (Two Question)

Game theory : Definition, characteristics of games, two person, zero sum game, pay of matrix strategy & its types, Saddle point, solution of rectangular game with saddle point, solution method of rectangular game in terms & strategy, strategy of mixed optimal strategy, concept of Dominance, Graphical method of solving (2xn) and (mx2) games, Algebraic method for the solution of general game, equivalence of the rectangular matrix games and linear programming, fundamental theory of game theory, limitation of game theory, solution of rectangular game by singular method, matrix method for (nxn) games.

Section –III (Two questions)

Nonlinear Programming-Definition and examples of non-linear programming, Mobi-Tucker theory: Kuhn-Tucker (K-T) optimality conditions, K-T first order necessary optimality conditions, K-T, second order optimality conditions, Lagrange's method, Economic interpretation of multipliers-Wolf duality theorem on non-linear programming, Quadratic programming, K-T conditions for Quadratic programming problems, Wolf modified simplex method, Beale's method, separable, convex and non-convex programming.

Section –IV (Two questions)

Inventory model : classification of inventory models, Determinsitc inventory model (DIM), Basic Economic-order quantity (EOQ) models, EOQ model with uniform rate of demand infinite production rate and having no shortage EOQ model with uniform rate of demand in different production cycles, infinite production rate & having non shortage, EOQ with finite replenishment DIM with shortage, Fixed Time Model, EOQ with finite production, EOQ with price break, EOQ with one price break, single multi-item

deterministic inventory model, Queuing models: classification of queuing models,
solution of queue models, model I (M/M/1) : (8/FCFS) model II (General Erlong
queuing model, model III M/M/1): (N/FCFS). Network (PERT/CPM), schedule chart
(Gantt Bar Chart), difference between CPM and PERT, Network components,
construction of the Network diagram, CPM analysis.

Books :

1. G.Hadley : Linear Programming
2. C.W. Churchman et.al. : Introduction to Operations Research
3. B.S. Goel & S.K. Mittal : Operations research
4. D. Gross & C.M. Harris : Fundamentals of Queuing Theory
5. A.O. Allen : Probability Statistics & Queuing Theory with
Computer
Science Applications

SEMESTER-IV

MM-511 (opt. iv) Fuzzy Sets and Applications-II

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80

+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two
questions from each section and one compulsory question. The compulsory
questions will consist of eight parts and distributed over the whole syllabus. The
candidate is required to attempt five questions selecting at least one from each
section and the compulsory question.

SECTION-I (Two questions)

Possibility Theory : Fuzzy measures, continuity from below and above, semicontinuous
fuzzy measures, examples and simple properties; Evidence Theory, belief measure,
superadditivity, monotonicity, plausibility measure, subadditivity, basic assignment, its
relation with belief measure and plausibility measure, focal element of basic assignment,
body of evidence, total ignorance, Dempster`s rule of combination, examples; Possibility
Theory, necessity measure, possibility measure, implications, possibility distribution function, lattice of possibility distributions, joint possibility distribution.

Fuzzy sets and possibility theory, degree of compatibility, degree of possibility, relation
with possibility distribution function and possibility measure, example of possibility
distribution for fuzzy proposition. Possibility theory versus probability theory,
characterization of relationship between belief measures and probability measures,
probability distribution function, joint probability distribution function, marginal
probability distributions, noninteractive, independent marginal distributions (Scope as in
the relevant parts of Chapter 7 of the book mentioned at the end.)

SECTION-II (Two questions)

Fuzzy Logic: An overview of classical logic, about logic functions of two variables,
Multivalued logics, about three-valued logic, n-valued logic, degrees of truth, definition
of primitives, Fuzzy propositions, classification, canonical forms, relation with possibility
distribution function, Fuzzy Quantifiers, their two kinds, relation with possibility

distribution function, Linguistic hedges, as a unary operation and modifiers, properties, Inference from conditional fuzzy propositions, relations with characteristic and membership functions, Compositional rule of inference, modus ponens and tollens, hypothetical syllogism, inference from conditional and qualified propositions, equivalence of the method of truth-value restrictions to the generalized modus ponens. (Scope as in the relevant parts of sections 8.1 to 8.7 of Chapter 8 of the book mentioned at the end.)

SECTION-III (Two questions)

Approximate reasoning: An overview of fuzzy expert system, Fuzzy implications as functions and operators, S-implications, R-implications, Gödel implication, QL-implications, Zadeh implication, examples, properties, combinations, axioms of fuzzy implications and characterization (only statement).
Selection of fuzzy implications, selection of approximate fuzzy implications to reasoning with unqualified fuzzy propositions, relation with compositional rule of inference, modus ponens and tollens,hypothetical syllogism Multiconditional approximate reasoning, method of interpolation, an illustration of the method for two if-then rules, as special case of compositional rule of inference and related results of fuzzy sets involved, The role of fuzzy relation equations, necessary and sufficient condition for a solution of the system of fuzzy relation equations for a fuzzy relation, its implications.(Scope as in the relevant parts of sections 11.1 to 11.5 of Chapter 11 of the book mentioned at the end .)

SECTION-IV (Two questions)

An introduction to fuzzy control: Fuzzy controllers, its modules, Fuzzy rule base, Fuzzy inference enzine, fuzzification and defuzzifications, steps of design of fuzzy controllers, defuzzification method, center of area method, center of maxima method and mean of maxima method. (Scope as in the relevant part of section 12.2 of chapter 12 of the book mentioned at the end .)

Decision –making in Fuzzy environment: Individual decision-making, fuzzy decision, simple examples, idea of weighting coefficients, Multiperson decision-making, fuzzy group decision, examples, Multicriteria decision-making, matrix representation of fuzzy relation, convertion to single-criterion decision, examples, Multistage decision-making, idea of principle of optimality, Fuzzy ranking methods, Hamming distance, priority set, examples, Fuzzy linear programming, two different methods one with only one side involving fuzzy numbers and other where only the coefficients of constraint matrix are fuzzy numbers . (Scope as in the relevant parts of Chapter 15 of the book mentioned at the end.)

Book :

G. J. Klir and B. Yuan : Fuzzy Sets and Fuzzy Logic Theory and Appplications.

Semester-IV

Paper MM-512 : Practical-IV

Time : 4 hours

Max. Marks : 100
Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-507 to MM-511 will be taught.

Part-B : Problem solving through MATLAB

Computer programs based on following Numerical Methods:

1. Solutions of simultaneous linear equations.
2. Solution of algebraic / transcendental equations.
3. Inversion of matrices
4. Numerical differentiation and integration
5. Solution of ordinary differential equations
6. Statistical problems on central tendency and dispersion
7. Fitting of curves by least square method.

Note :-Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee’s performance in written solutions/programs, execution of computer programs and viva-voce examination.

THE UNIVERSITY OF BURDWAN

RAJBATI, BURDWAN

WEST BENGAL

DETAILED SYLLABUS
SEMESTER-I
Paper – MCG101
(Functional Analysis-I & Real Analysis-I)
Unit-1

Functional Analysis-I

Total Lectures : 40 (Marks – 30)

Baire category theorem. Normed linear spaces, continuity of norm function, Banach spaces, Spaces $\mathbf{C}^n$, C [a,b] (with supmetric) , c_0, l_p $(1 \leq p \leq \infty)$ etc; (10L)

Linear operator, boundedness and continuity, examples of bounded and unbounded linear operators. (10L)

Banach contracton Principle – application to Picard's existence theorem and Implicit function theorem. (8L)

Inner product, Hilbert spaces, examples such as l_2 spaces, L_2[a,b] etc; C-S inequality, Parallelogram law, Pythagorean law, Minkowski inequality, continuity and derivatives of functions from $\mathbf{R}^m$ to $\mathbf{R}^n$. (12L)
6

Unit-2

Real Analysis-I

Total Lectures : 25 (Marks – 20)

Monotone functions and their discontinuities, Functions of bounded variation on an interval, their properties, Riemann-Stieltjes integral, existence, convergence problem and other properties. (12L)

Lebesgue outer measure, countable subadditivity, measurable sets and their properties, Lebesgue measure, measurable functions, equivalent functions, continuity and measurability, monotonocity and measurability, operation on collection of measurable functions, pointwise limit of a sequence of measurable functions, measurability of Supremum and Infimum, simple function and measurable function. (13L)

References

1. I. P. Natanson – *Theory of Functions of a Real Variable*, Vol. I, Fedrick Unger Publi. Co., 1961.

2. Lusternik and Sovolev-*Functional Analysis*

3. A.H. Siddiqui- *Functional Analysis with applications*, TMG Publishing Co. Ltd, New Delhi

4. K.K. Jha- *Functional Analysis, Student's Friends*,1986

5. Vulikh- *Functional Analysis*

6. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966

7. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958

8. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989

9. L.V. Kantorvich and G.P. Akilov- *Functional Analysis*, Pergamon Press,1982

10. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994

11. G.F. Simmons- *Introduction to Topology and Modern Analysis* ,Mc Graw Hill, New York, 1963

12. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd

13. Burkil & Burkil – *A second Course of Mathematical Analysis, CUP, 1980.*

14. Goldberg – *Real Analysis, Springer-Verlag, 1964*

15. Royden – Real Analysis, PHI, 1989

16. Lahiri & Roy – *Real Analysis, World Press, 1991.e*

17. Apostol – *Mathematical Analysis, Narosa Publi. House, 1985.*

18. Titchmarsh – *Theory of Functions, CUP, 1980*

19. W. Rudin- *Principle of Mathematical Analysis, Mc Graw Hill, Student Edn.*

20. Charles Swartz: Measure, Integration and Function Spaces, World Scientific, 1994.

Paper – MCG102
(Linear Algebra & Modern Algebra-I)
Unit-1
Linear Algebra
Total Lectures : 40 (Marks – 30)

Vector spaces, Euclidean space, Unitary space, orthonormal basis, Gram-Schmidt orthogonalization process. (8L)
Linear transformation in finite dimensional spaces, matrix of linear, rank and nullity, annihilator of a subset of a vector space. (5L)
Eigen vectors, spaces spanned by eigen vectors, similar and congruent matrices, characteristic polynomial, minimal polynomial, diagonalization, diagonalization of symmetric and Hermitian matrices, Cayley-Hamilton theorem, reduction of a matrix to normal form, Jordan Canonical form. (17L)
Quadratic form, Reduction to normal form, Sylvester's law of inertia, simultaneous reduction of two quadratic forms, applications to Geometry & Mechanics. (10L)

Unit-2
Modern Algebra -I
Total Lectures : 25 (Marks – 20)

Groups: Homomorphism, Isomorphism of groups, First and second isomorphism theorems, automorphisms and automorphism group, Inner automorphisms, groups of order 4 and 6, Normal sub groups and correspondence theorem for groups, simple groups. (10L)
Rings : Ring, commutative rings with identity, Prime & irreducible elements, division ring, Quaternions, idempotent element, Boolean ring, ideals, Prime ideal, maximal ideal, Isomorphism theorems, relation between Prime and maximal ideal, Euclidean domain, Principal ideal domain, Unique factorization domain, Polynomial rings. (15L)

References:

1. I. N. Herstein – *Topics in Algebra* (Vikas).

2. P. B. Bhattacharya, S. K. Jain & S. R. Noyapal – *Basic Abstract Algebra* (Cambridge)

3. T. W. Hungerford – Algebra (Springer).

4. Malik, Mordeson & Sen – *Fundamentals of Abstract Algebra (Tata MaGraw-Hill)*

5. *Sen, Ghosh & Mukhopadhyay – Topics in Abstract Algebra (University Press).*

6. *P. M. Cohn – Basic Algebra.*

7. *S. Lang – Algebra.*

8. *S. Lang – Linear Algebra.*

9. Hoffman & Kunze – Linear Algebra (Prentice Hall).

10. S. Kumareson – Linear Algebra.

11. Rao & Bhimsankaran – Linear algebra.

Paper – MCG103

(Elements of General Topology & Complex Analysis-I)

Unit-1

Elements of General Topology

Total Lectures : 40 (Marks – 30)

Topological spaces; definition, open sets, closed sets, closure, denseness, neighbourhood, interior points, limit points, derived sets, basis, subbasis, subspace. (10L)

Alternative way of defining a topology using Kuratowski closure operators and neighbourhood systems. (5L)

Continuous functions, homeomorphism and topological invariants. (3L)

First and second countable spaces, Lindelöf spaces, separable spaces and their relationship.(8L)

Separation axioms: T_0, T_1, T_2, T_3, $T_{3\frac{1}{2}}$, T_4 spaces, their simple properties and their relationship. (8L)

Introduction to connectedness and compactness. (6L)

Unit-2

Complex Analysis -I

Total Lectures : 25 (Marks – 20)

Complex Integration, line integral and its fundamental properties, Cauchy's fundamental theorem, Cauchy's integral formula and higher derivatives, power series expansion of analytic functions.(14L)

Zeros of analytic functions and their limit points, entire functions, Liouville's theorem. Fundamental theorem of algebra.(6L)

Simply connected region and primitives of analytic functions, Morera's theorem. (5L)

References

1. Simmons – *Introduction to Topology & Modern Analysis*

2. Munkresh – *Topology*

3. W. J. Thron – *Topological Structures*

4. Joshi – *General Topology*

5. J. L. Kelley – *General Topology*

6. J. B. Conway – *Functions of one Complex Variable* [Narosa]

7. R. B. Ash – *Complex Variable* [A.P.]

8. Punoswamy – *Functions of Complex Variable*

9. Gupta & Gupta – *Complex Variable*

10. W. Churchil- *Theory of Functional of Complex variable*

11. E. T. Copson- Functions of Complex variable

12. Philips- Functions of Complex variable

10

Paper – MCG104

(Ordinary Differential Equations & Special Functions, Operations Research-I)

Unit-1

Ordinary Differential Equations & Special Functions

Total Lectures : 40 (Marks – 30)

Ordinary Differential Equations

First order system of equations: Well-posed problems, existence and uniqueness of the solution, simple illustrations. Peano's and Picard's theorems (statements only) (8L)

Linear systems, non-linear autonomous system, phase plane analysis, critical points, stability, Linearization, Liapunov stability, undamped pendulum, Applications to biological system and ecological system (12L).

Special Functions

Series Solution : Ordinary point and singularity of a second order linear differential equation in the complex plane; Fuch's theorem, solution about an ordinary point, solution of Hermite equation as an example; Regular singularity, Frobenius' method – solution about a regular singularity, solutions of hypergeometric, Legendre, Laguerre and Bessel's equation as examples.(10L)

Legendre polynomial : its generating function; Rodrigue's formula, recurrence relations and differential equations satisfied by it; Its orthogonality, expansion of a function in a series of Legendre Polynomials.(6L)

Adjoint equation of n-the order: Lagrange's identity, solution of equation from the solution of its adjoint equation, self-adjoint equation, Green's function.(4L)

Unit-2

Operations Research-I

Total Lectures : 25 (Marks – 20)

Introduction, Definition of O.R., Drawbacks in definition, Scope of O.R., O.R. and decision making, Application of O.R. in different sectors, Computer application in O.R.(3L)

Fundamental theorem of L.P.P. along with the geometry in n-dimensional Euclidean space (hyperplane, separating and supporting plane).(3L)

11

Standard forms of revised simplex method, Computational procedure, Comparison of simplex method and revised simplex method, Sensitivity analysis, Bounded variable method, The Primal Dual Method.(14L)

Mathematical formulation of Assignment Problem, Optimality condition, Hungarian method, Maximization case in Assignment problem, Unbalanced Assignment problem, Restriction on Assignment, Travelling salesman problem.(5L)

References :

1. Wagner – *Principles of Operations Research* (PH)

2. Sasievir, Yaspan, Friedman – *Operations Research*: *Methods and Problems* (JW)

3. J. K. Sharma – *Operations Research – Theory and Applications*

4. Taha – *Operations Research*

5. Schaum's Outline Series – *Operations Research*

6. Hillie & Lieberman – *Introduction to Operations Research*

7. Swarup, Gupta & Manmohan – *Operations Research*

8. J. C. Burkill – *The Theory of Ordinary Differential Equations* [Oliver & Boyd, London]

9. E. A. Codington and Levinson – *Theory of Ordinary Differential Equations* [TMH]

10. R.P. Agarwal & R. C. Gupta – *Essentials of Ordinary Differential Equations* [MGH]

11. G. F. Simmons - Di*fferential Equations* [TMH]

12. G. Birkhoff & G. Rota - *Ordinary Differential Equation* [Ginn]

13. E. D. Rainville – *Special Functions* [Macmillan]

14. I. N. Sneddon - *Special Functions of mathematical Physics & Chemistry* [Oliver & Boyd, London]

15. N. N. Lebedev - *Special Functions and Their Applications* [PH]

Paper – MCG105

(Principle of Mechanics-I & Numerical Analysis)

Unit-1

Principle of Mechanics-I

Total Lectures : 40 (Marks – 30)

Generalised co-ordinates: Degrees of freedom, Constraint, Principle of Virtual Work. Lagrangian formulation of Dynamics: Lagrange's equations of motion for holonomic and non-holonomic systems. Ignoration of coordinates. (10L)

Routh's processs for the ignoration of co-ordinates. Rayleigh's dissipation function. Calculus of variation and Euler-Lagrange differential equations. Brachistochrone problem. (10L)

Configuration space and system point. Hamilton's principle; Hamilton's cannocical equations of motion. Principle of energy.(10L)

Principle of least action, Canonical Transformations, Poisson Bracket. (10L)

Unit-2

Numerical Analysis

Total Lectures : 25 (Marks – 20)

Numerical Methods : Algorithm and Numerical stability. (2L)

Graffae's root squaring method and Bairstow's method for the determination of the roots of a real polynomial equation. (4L)

Polynomial Approximation : Polynomial interpolation; Errors and minimizing errors; Tchebyshev polynomials; Piece-wise polynomial approximation. Cubic splines; Best uniform approximations, simple examples. (4L)

Richardson extrapolation and Romberg's integration method; Gauss' theory of quadrature. Evaluation of singular integral. (4L)

Operators and their inter-relationships : Shift, Forward, Backward, Central differences; Averaging operators, Differential operators and differential coefficients.(2L)

Initial Value Problems for First and Second order O.D.E. by

(i) 4th order R – K method

(ii) RKF_4 - method

(iii) Predictor – Corrector method by Adam-Bashforth, Adam-Moulton and Milne's method. (3L)

Boundary value and Eigen-value problems for second order O.D.E. by finite difference method and shooting method. (3L)
Elliptic, parabolic and hyperbolic P.D.E. (for two independent variables) by finite difference method; Concept of error, convergence & numerical stability. (3L)

References :

1. F. Chorlton – *A Text Book of Dynamic*

1. Synge and Griffith – *Principles of Mechanics*
2. D. T. Green Wood – *Classical Dynamics*
3. E. T. Whittaker – *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*
4. K. C. Gupta – *Classical Mechanics of Particles and Rigid Bodies*
5. F. Gantmacher – *Lectures in Analytical Mechanics*
6. H. Goldstein – *Classical Mechanics*
7. F. B. Hildebrand – *Introduction to Numerical Analysis*
8. Demidovitch and Maron – *Computational Mathematics*
9. F. Scheid – *Computers and Programming* (Schaum's series)
10. G. D. Smith – *Numerical Solution of Partial Differential Equations* (Oxford)
11. Jain, Iyengar and Jain – *Numerical Methods for Scientific and Engineering Computation*
12. A. Gupta and S. C. Basu – *Numerical Analysis*
13. Scarborough – *Numerical Analysis*
14. Atkinson – *Numerical Analysis*
15. Raulstan – *Numerical Analysis*

SEMESTER-II

Paper – MCG201

(Complex Analysis-II & Real Analysis-II)

Unit-1

Complex Analysis-II

Total Lectures : 40 (Marks – 30)

Open mapping theorem.(5L)
Singularities. Laurent's series expansion and classification of isolated singularities, essential singularities and Casorati-Weierstrass's theorem. Cauchy's residue theorem and evaluation of improper integrals.(12L)
Argument principle, Rouche's theorem and its application.(5L)
Maximum modulus theorem.(3L)
Conformal mappings, Schwarz's Lemma and its consequence.(10L)
Introduction to Analytic continuation.(5L)

Unit-2

Real Analysis-II

Total Lectures : 25 (Marks – 20)

Lebesgue integral of a simple function, Lebesgue integral of a non-negative (bounded or unbounded) measurable function, Integrable functions and their simple properties, Lebesgue integral of functions of arbitrary sign, Integrable functions, basic properties of the integral,

Integral of point wise limit of sequence of measurable functions- Monotone convergence theorem and its consequences, Fatou's lemma, Lebesgue dominated convergence theorem. Comparison of Lebesgue's integral and Riemann integral, Lebesgue criterion of Riemannian integrability. (17L)

Fourier series, Dirichlet's kernel, Riemann-Lebesgue theorem, Pointwise convergence of Fourier series of functions of bounded variation. (8L)

1. J. B. Conway – *Functions of one Complex Variable* [Narosa]

2. R. B. Ash – *Complex Variable* [A.P.]

3. Punoswamy – *Functions of Complex Variable*

4. Gupta & Gupta – *Complex Variable*

5. I. P. Natanson – *Theory of Functions of a Real Variable,* Vol. I

6. C. Goffman – *Real Functions*

7. Burkil & Burkil – *Theory of Functions of a Real Variable*

8. Goldberg – *Real Analysis*

9. Royden – Real Analysis

10. Lahiri & Roy – *Theory of Functions of a Real Variable*

11. Apostol – *Real Analysis*

12. Titchmarsh – *Theory of Functions*

13. Charles Scwarz-*Measure, Integration and Functions Spaces.*

Paper – MCG202

(Partial Differential Equations & Differential Geometry)

Unit-1

Partial Differential Equations

Total Lectures : 40 (Marks – 30)

General solution and complete integral of a partial differential equation; Singular solution; Integral surface passing through a curve and circumscribing a surface.(4L)

First order P.D.E, : Characteristics of a linear first order P.D.E.; Cauchy's problem; Solution of non-linear first order P.D.E. by Cauchys method of characteristics; Charpit's method (application only).(8L)

Second order linear P.D.E. : Classification, reduction to normal form; Solution of equations with constant coefficients by (i) factorization of operators, (ii) separation of variables; Solution of one-dimensional wave equation and diffusion equation; Solution of Laplace equation in Cylindrical and spherical polar co-ordinates. Formulation of Initial and Boundary Value Problem of P.D.E; Solution of Dirichlet's and Numann's problem of Laplace's equation for a circle.(28L)

Unit-2

Differential Geometry

Total Lectures : 25 (Marks – 20)

Reciprocal base system, Intrinsic derivative, Parallel vector field along a curve Space Curve, Serret – Frenet formula. (8L)

Metric tensor of the surface, angle between two curves lying on the surface, parallel vector field on a surface, Geodesics on a surface, Its differential equation, Geodesic curvature of a surface curve, Tensor derivative. (10L)

First fundamental form of the surface, Gauss's formula and second fundamental *form of the surface, Meusnier theorem and Euler's theorem. (7L)*

References:

1. T.Amarnath – *Partial Differential Equation*

2. I. N. Sneddon – *Partial Differential Equation*

3. H. Goldstein – *Classical Mechanics*

4. P. Phoolan Prasad & R. Ravichandan – *Partial Differential Equations*

5. C. E. Weatherburn – *Differential Geometry*

6. M. Postrikov – *Lectures in Geometry, Linear Algebra and Differential Geometry*

7. U. C. De- *Differential Geometry of Curves and Surfaces in E3, Anamaya Publi., 2007.*

8. M. P. Do Carmo- *Differential Geometry of Curves and Surfaces*

9. B. O'Neill- *Elementary Differential Geometry*

10. Rutter- *Geometry of Curves*

11. Andrew Pressely- *Elementary Differential Geometry*

Paper – MCG203

(Operations Research-II & Principle of Mechanics-II)

Unit-1

Operations Research-II

Total Lectures : 40 (Marks – 30)

Deterministic Inventory control Models: Introduction, Classification of Inventories, Advantage of Carrying Inventory, Features of Inventory System, Deterministic inventory models including price breaks.(14L)

Standard form of Integer Programming, The concept of cutting plane, Gomory's all integer cutting plane method, Gomory's mixed integer method, Branch and Bound method.(10L)

Processing of n jobs through two machines, The Algorithm, Processing of n jobs through m machines, Processing of two jobs through m machines.(6L)

Project scheduling by PERT/CPM : Introduction, Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Critical Path analysis, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation.(10L)

Unit-2

Principle of Mechanics-II

Total Lectures : 25 (Marks – 20)

Theory of small oscillations. Normal co-ordinates. Euler's dynamical equations of motion of a rigid body about a fixed point. Torque free motion. Motion of a top on a perfectly rough floor. Stability of top motion. Motion of a particle relative to rotating earth. Foucault's pendulum.(20L)

Special Theory of Relativity : Postulates; Special Lorentz Transformation ; Fitz-Gerald contraction and time-dilation. Einstein's velocity addition theorem. Relativistic mechanics of a particle, Energy equation $E = mc^2$*. (5L)*

References :

1. F. Chorlton – *A Text Book of Dynamic*

2. Synge and Griffith – *Principles of Mechanics*

3. D. T. Green Wood – *Classical Dynamics*

4. E. T. Whittaker – *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*

5. K. C. Gupta – *Classical Mechanics of Particles and Rigid Bodies*

6. I. S. Sokolnikoff – Mathematical Theory of Elasticity

7. Merovitch- *A treatise on dynamics*

8. Wagner – *Principles of Operations Research* (PH)

9. Sasievir, Yaspan, Friedman – *Operations Research*: *Methods and Problems* (JW)

10. J. K. Sharma – *Operations Research – Theory and Applications*

11. Taha – *Operations Research*

12. Schaum's Outline Series – *Operations Research*

13. Hillie & Lieberman – *Introduction to Operations Research*

14. Swarup, Gupta & Manmohan – *Operations Research*

Paper – MCG204

(Computer Programming & Continuum Mechanics-I)

Unit-1

Computer Programming

Total Lectures : 40 (Marks – 30)

Structured Programming in FORTRAN – 77: Subscripted variables, Type declaration, DIMENSION, DATA, COMMON, EQUIVALENCE, EXTERNAL statements. Function and subroutine sub – programs; Programs in FORTRAN – 77 (12L)

Programming in C: Introduction, Basic structures, Character set, Keywords, Identifiers, Constants, Variable-type declaration, Operators : Arithmetic, Relational, Logical, assignment, Increment, decrement, Conditional. (13L)

Operator precedence and associativity, Arithmetic expression, Evaluation and type conversion, Character reading and writing, Formatted input and output, Decision making (branching and looping) – Simple and nested IF, IF – ELSE, WHILE – DO, FOR. Arrays-one and two dimension, String handling with arrays – reading and writing, Concatenation, Comparison, String handling function, User defined functions. (15L)

Unit-2

Continuum Mechanics-I

Total Lectures : 40 (Marks – 30)

Continuous media, Deformation, Lagrangian and Eulerian approach; Analysis of strain (infinitesimal theory);.(10L)

Analysis of stress; Invariants of stress and strain tensors. Principle of conservation of mass; Principle of balance of linear and angular momentum; Stress equation of motion. (20L)

Necessity of constitutive equations. Hooke's law of elasticity, displacement equation of motion. Newton's law of viscosity (statement only). (10L)

References :

1. Ram Kumar – *Programming With Fortran –77*

2. P. S. Grover - *Fortran – 77/90*

3. Jain & Suri – *FORTRAN –77 Programming Language including FORTRAN – 90.*

4. G. C. Layek, A. Samad and S. Pramanik- *Computer Fundamentals, Fortran – 77 and Numerical Problems including C, S. Chand & Co.*

5. Xavier, C. – *C Language and Numerical Methods*, (New Age International (P) Ltd. Pub.)

6. *Gottfried, B. S. – Programming with C* (TMH).

7. Balaguruswamy, E. – *Programming in ANSI C* (TMH).

8. F. Scheid – *Computers and Programming* (Schaum's series)

9. T. J. Chang – *Continuum Mechanics* (Prentice – Hall)

10. Truesdell – *Continuum Mechanics* (Schaum Series)

11. Mollar – *Theory of Relativity*

12. F. Gantmacher – *Lectures in Analytical Mechanics*

13. J. L. Bansal – *Viscous Fluid Dynamics* (Oxford)

14. R. N. Chatterjee – *Contunuum Mechanics*

15. H. Goldstein – *Classical Mechanics*

Paper – MCGP205
Computer Aided Numerical Practical

Total Practical Classes : 65 (Marks – 50)
(Numerical Practical Using FORTRAN – 77)
[Problem – 25, Viva – 10 & Sessional – 15]
(Viva to be conducted on Paper – MG205 & MG206 only)

1. *Inversion of a non-singular square matrix. (6L)*

2. *Solution of a system of linear equations by Gauss – Seidel method. (4L)*

3. *Integration by Romberg's method. (5L)*

4. *Initial Value problems for first and second order O.D.E. by*

(i) Milne's method (First order) (6L)
(ii) 4th order Runge – Kutta method (Second order) (5L)

5. *Dominant Eigen – pair of a real matrix by power method (largest and least). (14L)*

6. *B.V.P. for second order O.D.E. by finite difference method and Shooting method. (5L)*

7. *Parabolic equation (in two variables) by two layer explicit formula and Crank–Nickolson – implicit formula. (12L)*

8. *Solution of one dimensional wave equation by finite difference method. (8L)*

References :

1. Ram Kumar – *Programming With Fortran –77*

2. P. S. Grover - *Fortran – 77/90*

3. Jain & Suri – *FORTRAN –77 Programming Language including FORTRAN – 90.*

4. G. C. Layek, A. Samad and S. Pramanik- *Computer Fundamentals, Fortran – 77 and Numerical Problems including C, S. Chand & Co..*

5. Xavier, C. – *C Language and Numerical Methods*, (New Age International (P) Ltd. Pub.)

6. *Gottfried, B. S. – Programming with C* (TMH).

7. Balaguruswamy, E. – *Programming in ANSI C* (TMH).

SEMESTER-III
PURE MATHEMATICS STREAM
Paper – MPG301
(Modern Algebra-II & Set Theory-I)
Unit-1
Modern Algebra-II
Total Lectures : 40 (Marks – 30)

Groups : *Direct product (internal and external), Group action on a set, Conjugacy classes and conjugacy class equation, p-groups, Cauchy's theorem, converse of Lagrange's theorem for finite commutative groups, Syllow theorems and applications, Normal series, solvable*

series, solvable and Nilpotent groups, Jordan-Holder Theorem, Finitely generated Abelian groups, Free Abelian groups. (20L)
..

Rings : *Unique factorization domain; Factorization of polynomials over a field; Maximal, Prime and primary ideal; Noetherian and Artinian Rings; Hilbert basis theorem. (20L)*

References :

1. I. N. Herstein – Topics In Algebra *(Wiley Eastern Ltd, New Delhi).*

2. M. Artin – Algebra *(P.H.I.).*

3. P. M. Cohn – Algebra, *Vol. – I. II, & III. (John Wiley & Sons).*

4. N. Jacobson – Basic Algebra *Vol. I, II.*

5. D. S. Malik, J. N. Mordeson & M. K. Sen – Fundamentals of Abstract Algebra *(McGraw – Hill, International Edition).*

6. J. B. Fraleigh – A First Course in Abstract Algebra *(Narosa).*

7. M. Gray – A Radical Approach to Algebra *(Addison Wesley Publishing Company).*

8. Hungerford – Algebra.

9. S. Lang – Algebra.

10. N. H. McCoy – The theory of Rings.

11. Burton – Ring Theory.

12. Gallian – Algebra.

Unit-2
General Topology-I
Total Lectures : 25 (Marks – 20)

Normal spaces, Urysohn's lemma and Tietze's extension theorem. (5L)
Product spaces, embedding lemma, Tychonoff spaces and characterization of Tychonoff spaces as subspaces of cubes. (6L)
Nets, filters subnets and convergence (4L)
Compactness, Compactness and continuity, countable compactness, sequential compactness, BW compactness and their relationship, Local compactness, Tychonoff

theorem (on Product of Compact Spaces) (10L)

References :

1. J. Dugundji – *Topology* (Allayn and Bacon, 1966)

2. J. L. Kelley – *General Topology* (Van Nostrand, 1955)

3. J. R. Munkres – *Topology – A First Course* (Prentice-Hall of India, 1978)

4. G. F. Simmons – *Introduction to Topology and Modern Analysis* (McGraw-Hill, 1963)

5. W. J. Thron – *Topological Structures* (Holt Reinhurt and Winston, 1966)

Paper – MPG302

(Graph Theory & Set Theory-I)

Unit-1

Graph Theory

Total Lectures : 55 (Marks – 40)

Graph, Subgraph, Complement, Isomorphism, Walks, Paths, cycles, connected components, Cut vertices and cut edges, Distance, radius and center, Diameter, Degree sequence, Havel-Hakimi Theorem (Statement only) (10L)

Trees, Centres of trees, Spanning trees, Eulerian and Semi Eulerian Graphs. Hamiltonian Graphs, Travelling Salesman Problem. (10L)

Vertex and edge connectivities, Blocks, Mengers Theorem. Clique Number, Independence number, Matching number, Vertex and edge conserving number, domination number, Ramsay's Theorem. (8L)

Chromatic number, Bipartite graph. Broke's Theorem, Mycielski Construction, Chromatic polynomial, edge colouring number, König Theorem. (6L)

Adjacency matrix, Incidence matrix, Cycle rank and co-cycle rank, Fundamental Cycles with respect to Spanning tree and Cayley's theorem on trees. (5L)

Planar graphs, Statement of Kuratowski Theorem, Isomorphism properties of graphs, Eulers formula, 5 colour theorem. Statement of 4 colour theorem, Dual of a planar Graph. (8L)

Directed Graph, Binary relations, directed paths, fundamental Circuits in Digraphs, Adjacency matrix of a Digraph. (8L)

References:

1. J. A. Bondy U.S.R. Murty – *Graph Theory with Applications* (Macmillan, 1976)

2. Nar Sing Deo – *Graph Theory* (Prentice-Hall, 1974)

3. F. Harary – *Graph Theory* (Addison-Wesley, 1969)

4. K. R. Pathasarthy – *Basic Graph Theory* (TMH., 1994).

Unit-2

Set Theory-I

Total Lectures : 15 (Marks – 10)

Axiom of choice, Zorn's Lemma, Hausdorff maximality principle, Well-ordering theorem and their equivalence, General Cartesian product of sets, Cardinal numbers and their ordering, Schröder-Bernstein theorem. (15L)

References :

1. K. Kuratowski – *Introduction to Set Theory and Topology*

2. E. Mendelson – *Introduction to Mathematical Logic*

3. R. R. Stoll – *Set Theory and Logic*

4. I. M. Copi – *Symbolic Logic*

5. W. Sierpienski – *Cardinal and Ordinal Numbers*

6. A. G. Hamilton – *Logic for Mathematicians* (Cambridge University Press)

Paper – MPG303
(Set Theory-II & Mathematical Logic, Functional Analysis-II)
Unit-1
Set Theory-II & Mathematical Logic
Total Lectures : 40 (Marks – 30)
Set Theory-II

Addition, multiplication and exponentiation of cardinal numbers, the cardinal numbers N_0 and C and their relation. (8L)

Totally ordered sets, order type, well-ordered sets, ordinal numbers, initial segments, ordering of ordinal numbers, addition and multiplication of ordinal numbers, sets of ordinal numbers, Transfinite induction. (7L)

Mathematical Logic

Statement calculus : Propositional connectives, statement form, truth functions, truth tables, Tautologies, contradiction, adequate sets of connectives (10L)

Arguments : Proving validity rule of conditional proof. Formal statement calculus, Formal axiomatic theory L, Deduction theorem (8L)

Consequences. Quantifiers, Universal and existential; symbolizing everyday language. (7L)

References :

1. K. Kuratowski – *Introduction to Set Theory and Topology*

2. E. Mendelson – *Introduction to Mathematical Logic*

3. R. R. Stoll – *Set Theory and Logic*

4. I. M. Copi – *Symbolic Logic*

5. W. Sierpienski – *Cardinal and Ordinal Numbers*

6. A. G. Hamilton – *Logic for Mathematicians* (Cambridge University Press)

1. K. Kuratowski – *Introduction to Set Theory and Topology*

2. E. Mendelson – *Introduction to Mathematical Logic*

3. R. R. Stoll – *Set Theory and Logic*

4. I. M. Copi – *Symbolic Logic*

5. W. Sierpienski – *Cardinal and Ordinal Numbers*

6. A. G. Hamilton – *Logic for Mathematicians* (Cambridge University Press)

Unit-2
Functional Analysis-II
Total Lectures : 25 (Marks – 20)

Completion of Metric space. Equicontinuous family of Functions. Compactness in C[0,1] (Arzela-Ascoli's Theorem). Convex sets in linear spaces. (8L)

Properties of normed linear spaces. Finite dimensional normed linear spaces. Riesz's Lemma, and its application in Banach spaces. Convergence in Banach Spaces. Equivalent Norms and their properties. (10L)

Principle of Uniform Boundedness (Banch-Steinaus), Open Mapping theorem. Closed graph theorem., Extension of continuous linear mapping. (7L)

References :

1. Lusternik and Sovolev-*Functional Analysis*

2. A.H. Siddiqui- *Functional Analysis with applications*, TMG Publishing Co. Ltd, New Delhi

3. K.K. Jha- *Functional Analysis, Student's Friends*,1986

4. Vulikh- *Functional Analysis*

5. G. Bachman & L. Narici- *Functional Analysis,* Academic Press,1966

6. A.E. Taylor- *Functional Analysis,* John wiley and Sons, New York,1958

7. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989

8. L.V. Kantorvich and G.P. Akilov-*Functional Analysis*, Pergamon Press,1982

9. B.K. Lahiri-*Elements of Functional Analysis,* The world Press Pvt. Ltd., Kolkata, 1994

10. G.F. Simmons- *Introduction to Topology and Modern Analysis* ,Mc Graw Hill, New York, 1963

11. B.V. Limaye- *Functional Analysis,* Wiley Easten Ltd

Paper – MPS304
(Special Paper-I)
Total Lectures : 65 (Marks – 50)

A: Differential Geometry of Manifolds-I

Definition and examples of differentiable manifolds. Tangent spaces. Jacobian map. One parameter group of transformations. Lie derivatives. Immersions and imbeddings. Distributions. (32L)

Exterior algebra. Exterior derivative. (10L)

Topological groups. Lie groups and Lie algebras. Product of two Liegroups. One parameter subgroups and exponential maps. Examples of Liegroups. Homomorphism and Isomorphism. Lie transformation groups. General linear groups. (15L)

Principal fibre bundle. Linear frame bundle. Associated fibre bundle. Vector bundle. Tangent bundle. Induced bundle. Bundle homomorphisms. (8L)

References :

1. R. S. Mishra, *A course in tensors with applications to Riemannian Geometry*, Pothishala (Pvt.) Ltd., 1965.

2. R. S. Mishra, *Structures on a differentiable manifold and their applications*, Chandrama Prakashan, Allahabad, 1984.

3. B. B. Sinha, *An Introduction to Modern Differential Geometry,* Kalyani Publishers, New Delhi, 1982.

4. K. Yano and M. Kon, *Structure of Manifolds,* World Scientific Publishing Co. Pvt. Ltd., 1984.

5. U. C. De and A. A. Shaikh, Differential Geometry of Manifolds, Narosa Publishing House Pvt. Ltd., 2007.

B: Advanced Real Analysis-I

Upper and lower limits of real function and their properties. (5L)

Pointwise differentiation of functions of linear intervals. Derivates and derivatives. Measurability of derivates. Differentiation of real functions. Dini derivates and their properties. Monotonicity theorem. Example of a continuous nowhere differentiable function. (30L)

Vitali's covering theorem in one dimension. Differentiability of monotone functions. Absolutely continuous functions and singular functions. Cantor ternery set and Cantor function. Indefinite Lebesgue Integral. Fundamental theorem of integral calculus for Lebesgue integral. Lusin's condition (N). Characterization of absolutely continuous functions (Banach-Zaricki theorem). (30L)

References :

1. Hewitt and Stormberg – *Real and Abstract Analysis*
2. H. L. Royden – *Real Analysis*
3. Saks – *Theory of Integrals*
4. W. Rudin – *Real and Abstract Analysis*
5. M. E. Munroe – *Measure and Integration*
6. I. P. Natanson – Theory of Functions of a Real Variable, Vols. I & II.
7. E. W. Hobson – Theory of Functions of a Real Variable, Vols. I & II

C: Advanced Functional Analysis-I

Topological vector spaces, Local base and its properties, Separation properties, Locally compact topological vector space and its dimension. Convex Hull and representation Theorem, Extreme points, Symmetric sets, Balanced sets, absorbing sets, Bounded sets in topological vector space. Linear operators over topological vector space, Boundedness and

continuity of Linear operators, Minkowski functionals, Hyperplanes, Separation of convex sets by Hyperplanes, Krein-Milman Theorem on extreme points. (30L)

Locally convex topological vector spaces, Criterion for normability, Seminorms, Generating family of seminorms in locally convex topological vector spaces. Barreled spaces and Bornological spaces, Criterion for Locally convex topological vector spaces to be (i) Barreled and (ii) Bornological. (15L)

Strict convexity and Uniform convexity of a Banach space. Uniform Convexity of a Hilbert Space. Reflexivity of a uniformly convex Banach space, Weierstrass approximation theorem in C[a,b] (20L)

References :

1. W. Rudin-*Functional Analysis*, TMG Publishing Co. Ltd., New Delhi,1973
2. A.A. Schaffer-*Topological Vector Spaces*, Springer , 2nd Edn., 1991
3. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
4. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
5. Diestel- *Application of Geometry of Banach Spaces*
6. Narici & Beckerstein- *Topological Vector spaces*, Marcel Dekker Inc, New York and Basel,1985

7. G.F. Simmons- *Introduction to topology and Modern Analysis* ,Mc Graw Hill, New York, 1963

8. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958

9. Lipschitz-*General Topology*, Schaum Series

10. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3^{rd} Edn., 1990

11. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970

12. Holmes-*Geometric Functional Analysis and its Application*

13. J. Horvoth-*Topological Vector spaces and Distributions*, Addison-Wesley Publishing Co., 1966

14. C. Goffman and G. Pedrick-First *Course in Functional Analysis*, PHI, New Delhi,1987

15. R. E. Edwards- *Functional Analysis*, Holt Rinchart and Wilson, New York,1965

16. A. Wilansky-*Functional Analysis*, TMG Publishing Co.Ltd, New Delhi, 1973

D: Rings of Continuous Function-I

28

The ring C(X) of the real valued continuous function on a topological space X, its subrings, the subring $C^*(X)$, their Lattice structure, ring homomorphisms and lattice homomorphism.(15L)

Zero-sets cozero-sets, their unions and intersection, completely separated sets, C^* -embedding, Urysohn's extension theorem and C-embedding. Pseudocompactness and internal characterization of Pseudocompact spaces.(15L)

Ideals, Z-filters, maximal ideals, prime ideals, prime filters and their relation.(5L)

Completely regular spaces and the zero-sets, weak topologies determined by C(X) and $C^*(X)$. Stone-Čech's therem concerning adequacy of Tychonoff spaces X for investigation of C(X) and $C^*(X)$, compact subsets and C – embedding, locally compact spaces and their properties.(10L)

Convergence of Z – filters, cluster points, prime Z – filters and convergence and fixed Z-filters.(5L)

Fixed ideals and compactness, fixed maximal ideals of C (X) and $C^*(X)$, their characterizations, the residue class rings modulo fixed maximal ideals in C(X) and $C^*(X)$ and the field of reals. Relation between fixed maximal ideals in C(X) and $C^*(X)$. Compactness and fixed ideals.(10L)

P- spaces, P-points and their properties, characterization of P-spaces, properties of P-spaces.(5L)

References :

1. Richard E. Chandler, *Hausdorff Compactifications* (Marcel Dekker, Inc. 1976).

2. L. Gillmen and M. Jerison, *Rings of Continuous Functions* (Von Nostrand, 1960).

3. *Topological Structures* (Halt Reinhurt and Winston, 1966).

E: Theory of Rings And Algebras-I

Rings and Ideals : Definitions, Ideal. Quotient rings and homorphisms, The field of quotients, Minimal and maximal conditions, Primary decomposition, Polynomial rings. (15L)

The Classical Radical : Nilpotent ideals and the radical, The radical of related rings, Artinian & Noethrian rings, Direct sum decompositions, Ideals in semisimple rings, Matrix rings, The Wedderburn theorem. (20L)

Modules : Preliminaries, Direct sums and free modules, projective modules, Tensor products, field and matrix representations, Algebras. (20L)

The Jacoson Radical : Primitive rings, The Density Theorem, Structure theorems. (10L)

References :

1. Mary Gray – *A Radical Approach to Algebra* (Addison-Wesley Publishing Company).
2. Ernst – August Behrems (Translated by Clive Reis) – *Ring Theory* (Academic Press, New York, London).
3. Stanley Burris & H. P. Sankappanvar – *A Course in Universal Algebra* (Springer-Verlag, New York, Berlin)
4. L. H. Rowen – *Ring Theory* (Academic Press)
5. T. Y. Lam – *Noncommutative Rings* (Springer-Verlag)
6. N. Jacolson – *Basic Algebra –II*
7. I. N. Herstein – *Noncommutative Rings*
8. N. J. Divisky – *Rings and Radicals*
9. N. H. Mc Coy – The Theory of Rings
10. M. R. Adhikari – Groups, Rings, Modules and applications

F: Non Linear Optimization In Banach Spaces-I

Review of Weak Convergence in normed spaces, reflexivity of Banach spaces, Hahn-Banach theorem and partially ordered linear spaces. (20L)

Existence Theorems for Minimal Points – Problem formulation. Existence theorems. Set of minimal points. (15L)

Applications to approximations and optimal control problems. (15L)

Generalised Derivatives-Directional derivative. Gâteaux and Frechet derivatives. Subdifferential. Quasidifferential Clarke derivative. (15L)

References :

1. Johannes John – *Introduction to the Theory of Nonlinear Optimization* (Springer-Verlag, 1994)
2. V. Bartu and T. Precupanu – *Convexity and Optimization in Banach Spaces* (Editura Acad. Bucaresti, 1986).
3. A. V. Balakrishnan – *Applied Functional Analysis* (Springer-Verlag)

G: Harmonic Analysis-I

Basic properties of topological groups, subgroups, quotient groups and connected groups. Discussion of Haar Measure without proof on R, T, Z, and some simple matrix groups. $L^1(G)$ and convolution with special emphasis on $L^1(R)$, $L^1(T)$, $L^1(Z)$. Approximate identities. (20L)

Fourier series. Fejer's theorem. The classical kernels. Fejer's Poisson's and Dirichlet's summability in norm and point wise summability. Fatou's Theorem. The inequalities of Hausdorff and Young. (20L)

Examples of conjugate function series. The Fourier transform. Kernels of R. The Plancherel theorem on R Planchere measure on R, T, Z. Maximal ideal space of L^1 (R), L^1 (T) and L^1 (Z). (25L)

References :

1. Henry Helson - *Harmonic Analysis* (Hindustan Pub. Corp., 1994)

2. E. Hewitt and K. A. Ross – *Abstract Harmonic Analysis* Vol. I (Springer-Verlag, 1993)

3. Y. Katznelson – *An Introduction to Harmonic Analysis,* (John Wiley, 1968)

4. P. Koosis – *Introduction of H^p Spaces* (Cambridge Univ. Press).

5. R. R. Goldberg – *Fourier transforms*

6. T. Huissain – *Introduction to topological groups*

H: Applied Functional Analysis-I

Review of basic properties of Hilbert spaces. Convex programming-support functional of a convex set. Minkowski functional. Separation Theorem. Kuhn-Tucker Theorem. Minimax theorem. Farkas theorem. (20L)

Spectral theory of operators. Spectral Theory of compact operations. Operators on a separable Hilbert space. Krein factorization theorem for continuous kernels and its consequences. L_2 spaces over Hilbert spaces. (30L)

Multilinear forms. Analyticity Theorems. Non-linear Volterra operators. (15L)

References :

1. A. V. Balakrishnan- *Applied Functional Analysis,* Springer-Verlag.

2. Dunford and Schwartz-*Linear operators,* vol. 1 & 11.

3. S. G. Krein-Linear Differential Equations in a Banach space.

4. K. Yosida-Functional Analysis.

Paper – MPS305
(Special Paper-II)
Total Lectures : 65 (Marks – 50)

A: Measure and Integration-I

Algebra and σ-algebra of sets. Monotone class of sets. Borel sets. Fσ and Gδ sets. Countably additive set function. Measure on σ – algebra. Outer measure and measurability. Extension of measures. Complete measures and completion of a measure space. Construction of outer measures. Regular outer measure. Lebesgue Stieltjes measures and distribution function. Example of non-measurable sets (Lebesgue). (30L)

Measurable functions. Approximation of measurable functions by simple functions. Egoroff's Theorem. Lusin's Theorem. Convergence in measure. Integrals of simple functions. Integral of measurable functions. Properties of Integrals and Integrable functions. Monotone convergence theorem. Fatou's Lemma, Dominated convergence Theorem, Vitali convergence theorem. (35L)

References :

1. I. P. Rana – *Measure and Integration*

2. G. D. Barra – *Measure and Integration*

3. Hewitt and Stormberg – *Real and Abstract Analysis*

4. H. L. Royden – *Real Analysis*

5. Saks – *Theory of Integrals*

6. W. Rudin – *Real and Abstract Analysis*

7. M. E. Munroe – *Measure and Integration*

8. Taylor - *Measure and Integration*

9. I. P. Natanson – *Theory of Functions of a Real Variable*, Vols. I & II.

10. Charles Schwartz- Measure , Integration and Function spaces, World Scientific Publi., Singapore, 1994.

B: Operator Theory And Applications-I

Adjoint operators over normed linear spaces; their algebraic properties. Compact operators on normal linear spaces, Sequence of Compact operators, Compact extensions, Weakly compact-operators(10L)

operator equation involving compact operators, Fredholm alternative; Adjoint operators on Hilbert-spaces, Self-adjoint operators, their algebraic properties; Unitary operators, normal operators in Hilbert spaces, positive operators, their-sum, product; Monotone sequence of positive operators, square-root of positive operator, Projection operators. (20L)

Their sum and product; Idempotent operators, positivity norms of Projection operators; Limit of monotone increasing sequence of Projection operators. (35L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966

2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd

3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989

4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994

5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,Mc Graw Hill, New York, 1963

6. N. Dunford and J.T. Schwartz-*Linear Operators, Vol-1&II*, Interscience, New York,1958

7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rd Edn., 1990

8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970

9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958

10. L.V. Kantorvich and G.P. akilov- *Functional Analysis*, Pergamon Press,1982

11. Vulikh- Functional Analysis

12. J. Tinsley Oden& Leszek F. Dernkowicz- Functional Analysis, CRC Press Inc, 1996.

13. Lipschitz-General Topology, Schaum Series

C: Algebraic Topology-I

Homotopy : Definition and some examples of homotopies, homotopy type and homotopy equivalent spaces, retraction and deformation, H-space.

33

Category : Definitions and some examples of category, factor and natural transformation. (10L)

Fundamental group and covering spaces : Definition of the fundamental group of a space, the effect of a continuous mapping on the fundamental group, fundamental group of a product space, notion of covering spaces, liftings of paths to a covering space, fundamental groups of a circle, (20L)

Universal cover, its existence, calculation of fundamental groups using covering space. Projection space and torus, homomorphisms and automorphisms of covering spaces, deck transformation group, Borsuk – Ulam theorem for S^2, Brower fixed-point theorem in dimension 2. (35L)

References :

1. W. S. Massey – *Algebraic Topology*
2. W. S. Massey – *Singular Homology Theory*
3. E. H. Spanier – *Algebraic Topology*
4. B. Gray – *Homotopy Theory An Introduction to Algebraic Topology*
5. C. R. Bredon – *Geometry and Topology*

D: Lattice Theory-I

Introduction : Partially ordered sets, graphs, order isomorphism, Maximal minimal condition, Jordan-Dedekind chain condition, dimension function. (10L)

Definition of an algebra, Lattices as algebras, density principle, Lattices as partially ordered sets sublattices Ideals, complements, semicomplements, atoms, Irreducible and prime elements, Morphisims homomorphisims, ideals direct products. (20L)

Closure operation, Dedekind condition, Dedekind cuts. Completion, interval topology.(15L)

Distributive and modular lattices, modularity and distributivity criterion, distributive sublattices of modular lattices, transposed intervals, meet representation in modular and distributive lattices. (20L)

References :

1. *G. Szasz* – Lattice Theory
2. *G. Birkhoff* – Lattice Theory
3. *B. H. Arnold* – Logic and Boolean Algebra
4. *D. E. Rutherford* – Lattice Theory

34

E: Advanced Operations Research-I

Non-linear Optimization : Local and global minima and maxima, convex functions and their properties, Method of Lagrange multiplier. (8L)

Optimality in absence of differentiability, Slater constraint qualification, Karlin's constraint qualification, Kuhn-Tuckers Saddle point optimality conditions, Optimality criterion with differentiability and convexity, separation theorems, Kuhn-Tuckers sufficient optimality theorem. (10L)

Unconstrained Optimization : Search method : Fibonacci search, Golden Section search; Gradient Methods : Steepest descent Quasi-Newton's method, Davidon-Fletecher-Powell

method, Conjugate direction method (Fletecher-Reeves method). (15L)
Optimality conditions : Kuhn-Tucker conditions – non negative constraints (6L)
Quadratic Programming : Wolfe's Modified Simplex method, Beale's method (8L)
Separable convex programming, Separable Programming Algorithm. (6L)
Network Flow : Max-flow min-cut theorem, Generalized Max flow min-cut theorem, Linear Programming interpretation of Max-flow min-cut theorem, Minimum cost flows, Minflow max-cut theorem. (12L)

References:

1. G. Hadly – *Non-Linear and Dynamic Programming,* Addision –Wesley, Reading Mass.
2. G. Hadly – Linear Programming, Narosa Publishing House.
3. S. S. Rao –*Optimization theory and Applications,* Wiley Eastern Ltd., New Delhi.
4. O. L. Mangasarian – *Non-Linear Programming,* McGraw Hill, New York.
5. Luenberger – *Introduction to Linear and Non-Linear Programming*
6. S. Dano – *Non-Linear and Dynamic Programming*
7. H. A. Taha – *Operations Research – An Introduction,* Macmillan Publishing Co., Inc., New York.
8. Swarup, Gupta & Manmohan – *Operations Research*, Sultan Chand & Sons, New Delhi.
9. *N.S. Kambo- Mathematical Programming Techniques,* Affiliated East-West Press Pvt. Ltd., New Delhi
10. M. C. Joshi and K.M. Moudgalya, Optimization theory and Practice, Narosa Publishing House, New Delhi
11. C.R. Bector, S. Chandra and J. Dutta, Principles of optimization Theory, Narosa Publishing House, New Delhi

12. M. A. Bhatti, Practical Optimization Methods, Springer -Verlag

F: Geometric Functional Analysis-I

Convexity in Linear spaces. Convex functions Basic separation Theorems. Convexity, and orderings. Alternate formulation of the separation Principle. Some Application. Extremal sets. Locally convex spaces. (30L)
Convexity and Topology. Weak Topology. (35L)

References :

1. Holmes – *Geometric Functional Analysis and Its Applications*

G: Proximities, Nearnesses and Extensions of Topology Spaces-I

Čech closure operator, closure spaces, symmetric Čech closure operator, continuity, homeomorphisms and their properties. Linkage compact topological spaces and their relation with compact topological spaces in presence of regularity condition. Extensions of closure spaces, trace system, principal (strict) extensions, ordering of extensions. Representation of principal T_0 extension of a T_0 topological space with a given trace system. The set of principal T_0 extensions of a T_0 topological spaces is a partially ordered set and its consequence for the class of T_2 compactification of a Tychonoff space. (35L)

(Basic) proximities, induced closure operators, proximity spaces, proximal neighbourhoods, p-continuous functions and their properties. Lattice structure of the basic proximities compatible with a symmetric closure space. Clans, clusters and relation between them. Basic proximities are clan generated structures. Classification of basic proximities : Riesz (RI-) proximities, Lodato (LO-) proximities, Efremovič (EF-) proximities, their characterization and relation between them. (30L)

References :

1. S. A. Naimpully and B. D. Wanack – *Proximity Spaces(*Cambridge Track No. 59, Cambridge, 1970)

2. W. J. Thron – *Topological Structures*(Halt Reinhurt and Winster, 1966)

3. E. Čech – *Topological Structures*(English Transl. Wiley, New York, 1966)

H: Advanced Complex Analysis-I

36

Analytic function, the functions M(r) and A(r). Theorem of Borel and Caratheodary, Convex function and Hadamard three-circle theorem, Phragmen-Lindelof theorem. (20L)
Harmonic function, Mean value property, Maximum principle, Harmonic function on a disk, Hamaek's inequality, Dirichlet's problem. (15L)
Integral function, Poisson Jenson formula, construction of an integral function with given zeros –Weierstrass theorem, Jensen's inequality, order, exponent of convergence of zeros of an integral function, canonical product, genus, Hadamard's factorization theorem, Borel's theorems, Picard's first and second theorems. (30L)

References :

1. J. B. Conway – *Functions of One Complex Variable*

2. L. V. Ahlfors – *Complex Analysis*

3. W. Rudin – *Real and Complex Analysis*

4. E. C. Titchmarsh – *Theory of Functions*

5. E. T. Copson – *Function of a Complex Variable*

6. R. P. Boas – *Entire Functions*

7. H. Cartan – *Analytic Functions*

8. A. I. Markusevich - *Theory of Functions of a Complex Variables*, Vol. I & II.

9. M. Dutta and Lokenath Debnath – *Elliptic Functions*.

SEMESTER-IV
PURE MATHEMATICS STREAM
Paper – MPG 401
(Modern Algebra-III)
Total Lectures : 65 (Marks – 50)

37

Field Theory: *Extension of fields, Simple extensions, Algebraic and Transcendental extensions, Splitting fields, Algebraically closed fields, normal extension, separable extensions, Perfect field. (30L)*
Automorphism of fields, Galois field, Galois extension, Fundamental Theorem of Galois theory, primitive elements, Solution of polynomial equations by radicals, Insolvability of the general equation of degree five or more by radicals, Cyclotomic extensions, Ordered field, Valuation, Completion. (20 L)

Modules: *Artinian and Noetherian Modules, Fundamental Structure Theorem for finitely generated modules over a P.I.D. and its application to finitely generated Abelian groups. (15L)*

References :

1. *S. Lang* – Algebra *(P.H.I.)*

2. *Hungerford* – Algebra.

3. *D. S. Malik, J. N. Mordeson & M. K. Sen* – Fundamentals of Abstract Algebra *(McGraw – Hill).*

4. *I. T. Adamson* – Introduction to Field Theory *(Cambridge University Press)*

5. *S. Lang – Algebra.*

6. *M. M. Postnikov – Fundamentals of Galois' Theory.*

7. *Dommit & Foote – Abstract Algebra.*

Paper – MPG402

(General Topology-II & Functional Analysis-II)

Unit-1

General Topology-II

Total Lectures : 40 (Marks – 30)

Connectedness and charecterization of connected subsets, union of connected subsets. Connected subsets of the real line, local connectedness, components, structure of open sets in locally connected second countable spaces, connectedness of the product spaces (10L)

One-point Compactification, Stone-Čech compactification(without proof) (3L)

Compactness in metric spaces, Properties of Compact metric spaces (4L)

Urysohn's metrization theorem, Uniform structure, uniform topology, uniform spaces, uniform continuity, Cauchy filter, total boundedness, completeness and compactness. (13L)

38

Homotopy of paths, covering spaces, fundamental group. Definition of the fundamental group of the circle. (10L)

References :

1. J. Dugundji – *Topology* (Allayn and Bacon, 1966)

2. J. L. Kelley – *General Topology* (Van Nostrand, 1955)

3. J. R. Munkres – *Topology – A First Course* (Prentice-Hall of India, 1978)

4. G. F. Simmons – *Introduction to Topology and Modern Analysis* (McGraw-Hill, 1963)

5. W. J. Thron – *Topological Structures* (Holt Reinhurt and Winston, 1966)

6. B. K. Lahiri- Algebraic topology, Narosa publishing House Pvt. Ltd., New Delhi.

Unit-2

Functional Analysis-II

Total Lectures : 25 (Marks – 20)

Invertible Mappings and their properties. Linear functionals. Hahn-Banach theorem and its applications. Conjugate spaces. Reflexive spaces (6L)

Properties of strong and weak convergence. Adjoint (Conjugate) operators and their properties. Hilbert Spaces, $L_p[a, b]$ $(1 \le p \le \propto)$. (5L)

Continuity of inner product. Convergence, Orthogonality and orthogonal decomposition of a Hilbert Space, Orthogonal set. Bessel's inequality. Perseval's identity. Minimization of norm problem. Complete orthonormal set.

Riesz-Fischer theorem. Riesz representation theorem for bounded linear functionals on Hilbert spaces. (14 L)

References :

1. Lusternik and Sovolev-*Functional Analysis*
2. A.H. Siddiqui- *Functional Analysis with applications*, TMG Publishing Co. Ltd, New Delhi
3. K.K. Jha- *Functional Analysis, Student's Friends*,1986
4. Vulikh- *Functional Analysis*
5. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
6. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
7. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
8. L.V. Kantorvich and G.P. Akilov-*Functional Analysis*, Pergamon Press,1982
9. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994

10. G.F. Simmons- *Introduction to Topology and Modern Analysis* ,Mc Graw Hill, New York, 1963
11. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd

Paper – MPS403

(Special Paper-III)

Total Lectures : 65 (Marks – 50)

A: Differential Geometry of Manifolds-II

Riemannian manifolds. Riemannian connection. Curvature tensors. Sectional Curvature. Schur's theorem. Geodesics in a Riemannian manifold. Projective curvature tensor. Conformal curvature tensor. (30L)

Submanifolds and Hypersurfaces. Normals. Gauss' formulae. Weingarten equations. Lines of curvature. Generalized Gauss and Mainardi-Codazzi equations. (10L)

Almost Complex manifolds. Nijenhuis tensor. Contravariant and covariant almost analysis vector fields. F-connection. (15L)

References :

1. R. S. Mishra, *A course in tensors with applications to Riemannian Geometry*, Pothishala (Pvt.) Ltd., 1965.

2. R. S. Mishra, *Structures on a differentiable manifold and their applications*, Chandrama Prakashan, Allahabad, 1984.

3. B. B. Sinha, *An Introduction to Modern Differential Geometry*, Kalyani Publishers, New Delhi, 1982.

4. K. Yano and M. Kon, *Structure of Manifolds*, World Scientific Publishing Co. Pvt. Ltd., 1984.

5. U. C. De and A. A. Shaikh, Differential Geometry of Manifolds, Narosa Publishing House Pvt. Ltd., 2007.

B: Advanced Real Analysis-II

Density of arbitrary linear sets. Lebsegue density theorem. Approximate continuity. Properties of approximately continuous functions. Bounded approximately continuous function over [a,b] and exact derivative. (15L)

The Perron integral : Definitions and basic properties, Comparison with Lebesgue integral and Newton integral. (10L)

Trigonomeric system and Trigonometric Fourier series. Summability of Fourier series by (C, I), means. Fejer's theorem. Lebesgue's theorem. Completeness of Trigonometric system. (15L)

Sets of the 1st and of the 2nd categories. Baires theorem for G_δ, residual and perfect sets, points of condensatia of a set. (10L)

Baire classification of functions. Functions of Baire class one. Baire’s theorem. Semicontinuous functions. (15L)

References :

1. Hewitt and Stormberg – *Real and Abstract Analysis*
2. H. L. Royden – *Real Analysis*
3. Saks – *Theory of Integrals*
4. W. Rudin – *Real and Abstract Analysis*
5. M. E. Munroe – *Measure and Integration*
6. I. P. Natanson – *Theory of Functions of a Real Variable*, Vols. I & II.
7. E. W. Hobson – *Theory of Functions of a Real Variable*, Vols. I & II

C: Advanced Functional Analysis-II

Stone-Weierstrass W-capital theorem in C(X,R) and C(X,C) where X is a compact Hausdorff space, Representation theorem for bounded linear functionals on C[a,b], l_p $(1 \leq p < \propto)$ and $L_p[a,b]$, $(1 \leq p < \propto)$, consequences of uniform boundedness principle, weak topology, weak* topology, Banach-Alaoglu theorem. (25L)

Approximation Theory in Normal Linear space, Best approximation, Uniqueness Criterion, Separable Hilbert Space. (15L)

Banach Algebra, Identity element, analytic property of resolvent Operator, Compactness of Spectrum, Spectral radius and Spectral mapping Theorem for polynomials, Gelfand Theory on representation of Banach Algebra, Gelfand Neumark Theorem. (25L)

41

References :

1. W. Rudin-*Functional Analysis*, TMG Publishing Co. Ltd., New Delhi,1973
2. A.A. Schaffer-*Topological Vector Spaces,* Springer , 2nd Edn., 1991
3. G. Bachman & L. Narici- *Functional Analysis,* Academic Press,1966
4. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
5. Diestel- *Application of Geometry of Banach Spaces*
6. Narici & Beckerstein- *Topological Vector spaces*, Marcel Dekker Inc, New York and Basel,1985
7. G.F. Simmons- *Introduction to topology and Modern Analysis* ,Mc Graw Hill, New York, 1963
8. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
9. Lipschitz-*General Topology*, Schaum Series
10. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rd Edn., 1990

11. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970

12. Holmes-*Geometric Functional Analysis and its Application*

13. J. Horvoth-*Topological Vector spaces and Distributions*, Addison-Wesley Publishing Co., 1966

14. C. Goffman and G. Pedrick-First *Course in Functional Analysis*, PHI, New Delhi,1987

15. R. E. Edwards- *Functional Analysis*, Holt Rinchart and Wilson, New York,1965

16. A. Wilansky-*Functional Analysis*, TMG Publishing Co.Ltd, New Delhi, 1973

D: Rings of Continuous Function-II

Partially ordered rings, convex ideals, absolutely convex ideals, properties of convex ideals, lattice ordered rings, total orderedness of the residue class rings modulo prime ideals in C(X) and $C^*(X)$, real ideals, hyper-real ideals in C(X). Limit ordinal, non-limit ordinals, compactness of the spaces of the ordinals, first uncountable ordinals space and its "one point compactification" and relation between the rings of continuous function on them, Characterization of real ideals.(20L)

Cluster point and convergence of Z–filters on a dense subset of a Tychonoff space. Characterization of C^*- embedded dense subset of a Tychonoff space. Construction of Stone-Čech compactification. More specific properties of βN and βQ and βR.(15L)

Characterization of maximal ideas in $C^*(X)$ and C(X). Gelfand-Kolmogorov theorem. Structure space of a commutative ring - another description of βX. The Banach-Stone theorem.(15)

Partial ordered set K(X) of the T_2 Compactifications of a Tychonoff space X, elements of K(X) and the subsets of $C^*(X)$. Local compactness and the complete lattice K(X).(15L)

References :

1. Richard E. Chandler, *Hausdorff Compactifications* (Marcel Dekker, Inc. 1976).

2. L. Gillmen and M. Jerison, *Rings of Continuous Functions* (Von Nostrand, 1960).

3. *Topological Structures* (Halt Reinhurt and Winston, 1966).

E: Theory of Rings And Algebras -II

Other Radicals and Radical Properties : The Levitzki Radical, Brown-Mcloy radicals, Amitsur's properties, Relations among the radicals. (15L)

Generalizations of the notions of radicals to other systems : Algebras, Group Algebras, Near rings, Groups, Lattices. (10L)

Lie & Jordan Algebras : Definitions, Nilpotenoy and Solvability, A structure theorem for nonassociative algebras, Jordan Algebras, Lie Algebras, Simple Lie and Jordan algebras. (20L)

Category Theory : Definition, Functions, Objects and morphisms, Kernels and images, Exact Categories, Products & limits, abelian Categories.

Radical subcategories, Applications of sheaf theory to the study of rings.

Elements of Universal Algebra. (20L)

References :

1. Mary Gray – *A Radical Approach to Algebra* (Addison-Wesley Publishing Company).

2. Ernst – August Behrems (Translated by Clive Reis) – *Ring Theory* (Academic Press, New York, London).

3. Stanley Burris & H. P. Sankappanvar – *A Course in Universal Algebra* (Springer-Verlag, New York, Berlin)

4. L. H. Rowen – *Ring Theory* (Academic Press)

5. T. Y. Lam – *Noncommutative Rings* (Springer-Verlag)

6. N. Jacolson – *Basic Algebra – II*I.

7. N. Herstein – *Noncommutative Rings*

8. N. J. Divisky – *Rings and Radicals*

9. N. H. Mc Coy – *The Theory of Rings*

10. M. R. Adhikari – *Groups, Rings, Modules and applications*

F: Non Linear Optimization In Banach Spaces-II

Tangent Cones-Definition and properties. Optimality Conditions. Lyusternik theorem.

Generalized Lagrange Multiplier Rule – Problem formulation. Necessary and Sufficient optimality conditions. Application to optimal control problems. (20L)

Duality-Problem formulation. Duality theorem. Saddle point theorems. Linear problems. Application to approximation problems. (15L)

Some special optimization problems-Linear quadratic optimal control problems. Time optimal control problems. (30L)

References :

1. Johannes John – *Introduction to the Theory of Nonlinear Optimization* (Springer-Verlag, 1994)

2. V. Bartu and T. Precupanu – *Convexity and Optimization in Banach Spaces* (Editura Acad. Bucaresti, 1986).

3. A. V. Balakrishnan – *Applied Functional Analysis* (Springer-Verlag)

G: Harmonic Analysis-II

Hardy spaces on the unit circle. Invariant subspaces. Factoring. Proof of the F. and M. Riesz theorem. Theorems of Beurings and Szego in multiplication operator form. Structure of inner and outer functions. (20L)

The Inequalities of Hardy and Hilbert. Conjugate functions. Theorems of Kolmogorov & Zygmund and M. Riesz & Zygmund on conjugate functions. (20L)

The conjugate function as a singular integral. Statement of theBurkholder-Gundy Silverstein Theorem on T. Maximal functions of Hardy and Littlewood Translation. The Theorems of Wiener and Beurling. The Titchmarsh Convolution Theorem. Wiener’s Tauberian Theorem. Spectral sets of bounded functions. (25L)

References :

1. Henry Helson - *Harmonic Analysis* (Hindustan Pub. Corp., 1994)

2. E. Hewitt and K. A. Ross – *Abstract Harmonic Analysis* Vol. I (Springer-Verlag, 1993)

3. Y. Katznelson – *An Introduction to Harmonic Analysis*, (John Wiley, 1968)

4. P. Koosis – *Introduction of H^p Spaces* (Cambridge Univ. Press).

5. R. R. Goldberg – *Fourier transforms*

6. T. Huissain – *Introduction to topological groups*

H: Applied Functional Analysis-II

Semigroups of linear operators-general properties of semigroups. Generation of semigroups. Dissipative semigroups. Compact semigroups. (20L)

Holomorphic semigroups, Elementary examples of semigroups. Extension. Differential Equations. Cauchy Problem, Controllability. State reduction. Ovservability. Stability and stabilizability. Evolution equations. (30L)

Optimal Control Theory-Linear quadratic regulator problem. The same problem with infinite time interval. Hard constraints. Final value control. Time optimal control problems. (15L)

References :

1. A. V. Balakrishnan- *Applied Functional Analysis*, Springer-Verlag.

2. Dunford and Schwartz-*Linear operators*, vol. 1 & 11.

3. S. G. Krein-*Linear Differential Equations in a Banach space.*

4. K. Yosida-*Functional Analysis.*

Paper – MPS404
(Special Paper-IV)
Total Lectures : 65 (Marks – 50)

A: Measure and Integration-II

Signed measures, Hahn-decomposition theorem. Jordan decomposition theorem. Radon-Nidodym theorem. Radon-Nikodym derivative. Lebesgue decomposition theorem. Complex measure. Integrability of fuctions w.r.t. signed measure and complex measure. (30L)

Measurable Rectangles, Elementary sets. Product measures. Fubini's theorem. (20L)

L_p [a,b] – spaces ($1 \le p \le \infty$). Holder and Minkowski inequality. Completeness and other properties of L_p [a, b] spaces. Dense subspaces of L_p [a, b] – spaces. Bounded linear functionals on L_p [a, b] – spaces and their representations. (15L)

References :

1. I. P. Rana – *Measure and Integration*

2. G. D. Barra – *Measure and Integration*

3. Hewitt and Stormberg – *Real and Abstract Analysis*

4. H. L. Royden – *Real Analysis*

5. W. Rudin – *Real and Abstract Analysis*

6. M. E. Munroe – *Measure and Integration*

7. Taylor - *Measure and Integration*

8. I. P. Natanson – *Theory of Functions of a Real Variable*, Vols. I & II.

9. Charles Schwartz- Measure, Integration and Functions Spaces, World Scientific Publi., 1994.

B: Operator Theory And Applications-II

Spectral properties of bounded-Linear operators in normed linear space; Spectrum, regular value, resolvant of operator; closure property and boundedness property of spectrum, spectral radius. (20L)

Eigenvalues, eigen-vectors of self-adjoint operators in Hilbert space, Resolvant sets, real property of spectrum of self-adjoint operators, range of spectrum, Orthogonal direct sum of Hilbert space,(20L)

Spectral-theorem for compact normal operators, Sesquilinear functionals, property of boundedness and symmetry, Generalisation of Cauchy-Schwarz inequality. (15L)

Unbounded-operators and their adjoint in Hilbert spaces. (10L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966

2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd

3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989

4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994

5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,Mc Graw Hill, New York, 1963

6. N. Dunford and J.T. Schwartz-*Linear Operators, Vol-1&II*, Interscience, New York,1958

7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rd Edn., 1990

8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970

9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958

10. L.V. Kantorvich and G.P. akilov- *Functional Analysis*, Pergamon Press,1982

11. Vulikh- Functional Analysis

12. J. Tinsley Oden& Leszek F. Dernkowicz- Functional Analysis, CRC Press Inc, 1996.

13. Lipschitz-General Topology, Schaum Series

C: Algebraic Topology-II

Introduction of singular homology and cohomology group by Eilenberg and steenrod axioms. Existence and Uniqueness of singular homology and cohomology theory. (20L)

Calculation of homology and chomology groups for circle. Projective spaces, torus relation between $H_1(X)$ and $\pi_1(X)$. (20L)

Singular cohomology ring, calculation of cohomology ring for projective spaces. Fibre bundles : Definitions and examples of bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles and their morphisoms, cross sections, fibre products, induced bundles and vector bundles, homotopy properties of vector bundles. Homology exact sequence of a fibre bundle.(25L)

References :

1. W. S. Massey – *Algebraic Topology*

2. W. S. Massey – *Singular Homology Theory*

3. E. H. Spanier – *Algebraic Topology*

4. B. Gray – *Homotopy Theory An Introduction to Algebraic Topology*

5. C. R. Bredon – *Geometry and Topology*

D: Lattice Theory-II

Covering condition in modular lattice, modular lattices of locally finite length, Complemented modular lattices, Boolean algebras, complete Boolean algebras, Boolean algebras and Boolean rings, valuation of a lattice, metric and quasimetric lattice. (25L)

Complete Lattice, conditionally complete Lattices, Fix point theorem, Compactly generated lattices, subalgebra lattices. (20L)
Birkhoff lattices, Semimodular lattices, Complemented semimodular lattices, Ideal chains, Ideal lattices, Distributive lattices and ring of sets, Congruence relations, Ideals and congruence relations. (20L)

References :

1. G. Szasz – Lattice Theory

2. G. Birkhoff – Lattice Theory

3. B. H. Arnold – Logic and Boolean Algebra

4. D. E. Rutherford – Lattice Theory

E: Advanced Operations Research-II

Dynamic Programming : Characteristics of Dynamic Programming problems, Bellman's principle of optimality (Mathematical formulation)
Model –1 : Single additive constraint, multiplicative separable return,
Model – 2 : Single additive constraint, additively separable return,
Model – 3 : Single a multiplicative constraint, additively separable return,
Multistage decision process – Forward and Backward recursive approach, Dynamic Programming approach for solving linear and non-linear programming problems, Application – Single-item N-period deterministic inventory model. (25L)
Geometric Programming : Elementary properties of Geometric Programming and its applications. (8L)
Queuing Theory : Introduction, characteristic of Queuing systems, operating characteristics of Queuing system. Probability distribution in Queuing systems. Classification of Queuing models. Poisson and non-Poisson queuing models (32L)

References:

1 G. Hadly – *Non-Linear and Dynamic Programming,* Addision –Wesley, Reading Mass.

2 S. Dano – *Non-Linear and Dynamic Programming*

3 H. A. Taha – *Operations Research – An Introduction,* Macmillan Publishing Co., Inc., New York..

4 Swarup, Gupta & Manmohan – *Operations Research,* Sultan Chand & Sons, New Delhi.

5 N.S. Kambo- *Mathematical Programming Techniques*, Affiliated East-West Press Pvt. Ltd., New Delhi

F: Geometric Functional Analysis-II

Extreme points. Convex functions and optimization. Some More Applications: The category Theorems. (30L)

The Smulian Theorems. The theorem of James. Support Points and smooth points. Some further Application. Isomorphism of certain conjugate spaces. Universal spaces. (35L)

References :

1. Holmes – *Geometric Functional Analysis and Its Applications*

G: Proximities, Nearnesses and Extensions of Topology Spaces-II

Separated proximities, separation axioms satisfied by the closure operators induced by RI – (LO -, EF-) proximities. The Lattice structure of the class of RI – (LO-, EF-) proximities compatible with a suitable closure operator. (Basic) nearness, near families, contiguities, contigual families, closure operators, proximities and contiguities induced by a (basic) nearnesses, merotopic spaces. Nearness preserving maps. Separated nearnesses. The class of basic nearnesses compatible with a symmetric closure space (a proximity space, a contiguity space) and their Lattice structure. (30L)

Clans, clusters and cluster generated (concrete) nearnesses. Nearnesses are not clan generated structures. Classification of basic nearness : Riesz (RI -) nearnesses, Lodato (LO -) nearnesses and Efremovič (EF -) nearnesses, their characterization and relationship between them. Nearness spaces, cluster generated nearness spaces, contigual nearness spaces and proximal nearness spaces and relation between them. Correspondence between the principle (strict) T_1 extensions of a T_1 topological space X and the cluster generated compatible LO – nearnesses on X ; the correspondence between principal T_1 compactification of X and the compatible contigual LO – nearnesses on X ; the correspondence between the principal T_1 linkage compactifications of X and the compatible proximal LO – nearnesses on X . The correspondence between EF – nearnesses on a Tychonoff space X and the T_2 compactifications of X. (35L)

49

References :

1. S. A. Naimpully and B. D. Wanack – *Proximity Spaces(*Cambridge Track No. 59, Cambridge, 1970)

2. W. J. Thron – *Topological Structures*(Halt Reinhurt and Winster, 1966)

3. E. Čech – *Topological Structures*(English Transl. Wiley, New York, 1966)

H: Advanced Complex Analysis-II

Spaces of continuous functions, Ascoli-Arzela theorem, Spaces of Analytic functions, Hurwitz's theorem, Riemann mapping theorem. (20L)

Meromorphic function, Mittag-Leffler's theorem. (10L)

Elliptic function, weirstrass's elliptic function p(z), addition theorem for p(z),

differential equation satisfied by p(z), the numbers e_1, e_2, e_3. *(35L)*

References :

1. J. B. Conway – *Functions of One Complex Variable*

2. L. V. Ahlfors – *Complex Analysis*

3. W. Rudin – *Real and Complex Analysis*

4. E. C. Titchmarsh – *Theory of Functions*

5. E. T. Copson – *Function of a Complex Variable*

6. R. P. Boas – *Entire Functions*

7. H. Cartan – *Analytic Functions*

8. A. I. Markusevich - *Theory of Functions of a Complex Variables*, Vol. I & II.

9. M. Dutta and Lokenath Debnath – *Elliptic Functions*.

Paper – MPT405
Term Paper
Marks: 50

Term paper MPT405 is related with the special papers of the pure stream offered by the department in each session and the topic of the term paper will also be decided by the department in each session. However the mark distribution is 30 marks for written submission, 15 marks for seminar presentation and 5 marks for viva-voce.

SEMESTER-III
APPLIED MATHEMATICS STREAM
Paper – MAG301
(Methods of Applied Mathematics -I)
Methods of Applied Mathematics -I
Total Lectures : 65 (Marks – 50)
Integral Transforms

Fourier Transform and its properties, Inversion formula of F.T.; Convolution Theorem; Parseval's relation. Applications. Outline of Finite Fourier transform and its inversion formula. (10L)

Laplace's Transform and its properties. Inversion by analytic method and by Bromwitch path. Lerch's Theorem. Convolution Theorem; Applications. (10L)

Integral Equations

Linear Integral Equation, Reduction of differential equation to integral equation, Existence, Uniqueness and iterative solution of Fredholm and Volterra Integral equations; examples, Solution of Fredholm integral equation for degenerate kernel; Examples, Faltung type(closed cycle type) integral equation, Singular integral equation; Solution of Abel's integral equation. (20L)

Generalised Functions

Generalised function; Elementary properties; Addition, Multiplication, Transformation of variables. Generalized function as the limit of a sequence of good functions, Differentiation of generalized function. Simple examples, Antiderivative, Regularisation of divergent integral : Simple example, Fourier Transform of generalized function, Examples, Convergence of a sequence of generalized functions; Examples, Laplace transform of generalized function. (12L)

Operator Equations on Hilbert Spaces

Inner product spaces, Hilbert spaces; orthonormality; closedness, and completeness of sets, Fourier expansion, Reisz Fischer theorem. (Proof not reqd.). Isometric isomorphism between a separable Hilbert space and l_2, Linear operators on Hilbert space, continuity, boundedness, adjointness, self-adjointness, invertibility, boundedness and unboundedness of inverse.

Compactness, illustrative examples. Eigen value problems, Spectral theorem for compact self-adjoint operatiors, Application to Regular Sturm-Liouivile problem, Integral equations with Hilbert-Schmidt kernel, implications on Laplace operatior. Solvability of operator equations, Fredholm alternatives. (13L)

References :

1. Gelfand & Shilov – *Generalised Functions* (Academic Press)
2. I. N. Sneddon – *Fourier Transforms* (MacGraw-Hill)
3. R. V. Churchill – *Operational Methods*
4. Lusternik & Sobolev – *Functional Analysis*
5. Erwin Lareyizey – *Introductory Functional Analysis with Applications*
6. S. G. Mikhlin – *Integral Equation* (Pergamon Press)
7. F. G. Tricomi – *Integral Equation* (Interscience Publishers)
8. WE. V. Lovit. – *Linear Integral Equations* (Dover Publishers)
9. F. John – *Partial Differential Equations*
10. Williams - *Partial Differential Equations*
11. Epstein - *Partial Differential Equations*
12. Chester - *Partial Differential Equations*
13. Arnold – *Ordinary Differential Equatios*

Paper – MAG302

(Methods of Applied Mathematics –II, Theory of Electro Magnetic Fields)

Unit-1

Methods of Applied Mathematics -II

Total Lectures : 40 (Marks – 30)

Linear ordinary differential equations; generalized solution, fundamental solution, inverse of a differential operator. Two-point boundary value problem for a second-order linear O.D.E. Green's functions and its bilinear expansion, particular integral, Analogy between linear simultaneous algebraic equations and Linear differential equation. (6L)

Mathematical models and initial boundary value problems of 2^{nd} order partial differential equation (PDE); wellposedness; necessity of classification and canonical forms. Invariance of nature of an equation and its characteristics under coordinate transformation; transformation

of semilinear 2^{nd} order PDE in two independent variables; linear transformations, and linear PDE's with more than two independent variables. (5L)

Linear hyperbolic PDE's in two independent variables Cauchy problem. Cauchy-Kowalasky theorem (statement only) reason for restriction on cauchy ground curve. Rieman-Green function. Domain of dependence and influence. Possible discontunuites of solutions; d'Alemberts solution and meaning of generalized solution. (6L)

Linear parabolic equations : Heat equation in two independent variables, solution of Cauchy problem using Dirac Delta function and Fourier transform, maximum principle for initial – boundary value (for Dirichlet boundary condition) problem, uniqueness and stability of solution.

Methods of Eigen function empansion and Green's function; Separation of variables, formulation of eigenvalue problems related to wave, heat and Laplace equations. (8L)

Linear elliptic equation: Laplace equation : boundary value problems of Dirichlet, Neumann and Robin. Greens formulas involving Laplacian; mean value theorem, maximum principle, uniqueness and stability of solutions; Dirichlet principle, Rayleigh-Ritz method. (5L)

Greens function for Dirichlet problem on Laplace eqn. its properties and methods of construction. Method of images. Method of conformal mapping for 2-dimentional problem with problem of a unit circle as an example. Bilinear expansion for Green's funcion; Green's function for heat equation by the method of Eigen function expansion and Bilinear expansion for Dirac Delta function. (10L)

References :

1. G. F. C. Duff and D. Naylor – *Differential Equations of Applied Mathematics* (Wiley International).
2. Stakgold – *Greens Functions and Boundary Value Problems* (John Wiley & Sons.)
3. D. H. Griffet – *Applied Functional Analysis* (Ellis Horwood Ltd. John Wiley & Sons.)
4. V. S. Vladiminov – *Equations of Mathematical Physics* (Marcel Danker, Inc. N.Y.)
5. Tikhnov & Samarski – *Equations of Mathematical Physics*

Unit-2

Theory of Electro Magnetic Fields

Total Lectures : 25 (Marks – 20)

Empirical basis of Maxwell's Equations : Coulomb's law, Gauss' law, Electrostatic potential, Steady current, Equation of continuity of charge, Biot-Savart's law, Magnetic induction, Ampere's law, Faraday's law, Maxwell's equations for electromagnetic field and their empirical basis. Material equations, Conditions at an interface, Electromagnetic potentials, Electromagnetic energy, Poynting theorem. (15L)

Application of Maxwell's Equations :

Plane electromagnetic Waves in vacuo, dielectric and conducting media, Group velocity and phase velocity, Retarded and accelerated potentials, Reflection and Refraction of plane waves at the plane boundary between two dielectrics, Field of a point charge in uniform motion. (10L)

References :

1. *W. K. H. Panofsky & M. Phillips –* Classical Electricity and Magnetism *(Addison-Wesley Pub. Co. Inc., 1962).*
2. *J. R. Reitz & F. J. Milford –* Foundations of Electromagnetic Theory *(Addison-Wesley Pub. Co., 1966)*
3. *D. J. Griffth –* Classical Electrodynamics *(Wiley Eastern, 1965)*

Paper – MAG303

(Continuum Mechanics-II, Dynamical Systems)

Unit-1

Continuum Mechanics-II

Total Lectures : 40 (Marks – 30)

General : Body, configuration, axiom of continuity, Motion of a body. Reference *configuration, deformation. Material and spatial coordinates. Material and spatial time derivatives – relation between them. Velocity and acceleration. Physical components of acceleration in general curvilinear coordinates – in cylindrical coordinates and spherical polar coordinates, Deformation gradient tensor, Reynolds transport theorem for volume property. Principle of conservation of mass – Path lines, stream lines and streak lines. Material surface. (Bounding surface). Lagrange's criterion for material surface, Polar decomposition theorem (Statement), polar decomposition of deformation gradient tensor – stretch and rotation tensors, principal stretches. Classical theory of infinitesimal*

deformations. Compatibility equations, Relative deformation gradient tensor, relative stretch tensors and relative rotation tensor. Rate-of –strain tensor-its principal values and invariants rate-of rotation tensor – vorticity vector; velocity gradient tensor, General principles of momenta balance; Euler's laws of motion. Body forces and contact forces. Cauchy's laws of motion: Stress equation of motion and symmetry of stress tensor for non-polar material. Energy balance – first and second laws of thermodynamics. (20L)

Constitutive equation (stress-strain relations) for isotropic elastic solid. Elastic modulii. Strain-energy function. Beltrami-Michel compatibility equations for stresses. Equations of equilibrium and motion in terms of displacement. Fundamental boundary value problems of elasticity and uniqueness of their solutions (statement only). Saint-Venant's principle – solution of simple problems. Wave propagation in an infinite elastic medium, Waves of distortion and dilatation. (20L)

References :

1. Leigh, D. C. – *Non-Linear Continuum Mechanics* (MacGraw-Hill)
2. Truesdell, C – *Continuum Mechanics*
3. Chung, T. J. – *Contunuum Mechanics* (Prentice-Hall)
4. Truesdell, C and Nol, W. – *Encyclopaedia of Physics.* Vol. III/3, 1965 (Ed. S. Flugge)
5. Sokolnikoff, I. S. – *Mathematical Theory of Elasticity*
6. Milne – Thomson, L. M.- *Theoretical Hydrodynamics*
7. Pai, S. L. – *Viscous Flow Theory*
8. Schilichting, H. – *Boundary Layer Theory*
9. Eriengen, A. C. – *Non-linear Theory of Continuous Media* (MacGraw-Hill)
10. F. Chorlton – *A Text Book of Fluid Mechanics*
11. Kolin, Kebel & Roze – *Theoretical Hydromechanics*
12. Besand & Ramsey – *Fluid Mechanics*
13. J. Bansal – *Viscous Flow Theory*

Unit-2

Dynamical Systems

Total Lectures: 25 Marks: 20

Dynamical Systems : *Phase variables and Phase space, continuous and discrete time systems, flows(vector fields), maps (discrete dynamical systems), orbits, asymptotic states,* *fixed (equilibrium) points periodic points, concepts of stability and SDIC (sensitive dependence of initial conditions) chaotic behaviour, dynamical system as a group. (6L)*

Linear systems : Fundamental theorem and its application. Properties of exponential of a matrix, generalized eigenvectors of a matrix, nilpotent matrix, stable, unstable and center subspaces, hyperbolicity, contracting and expanding behaviour. (6L)

Nonlinear Vector Fields : Stability characteristics of an equilibrium point. Liapunov and asymptotic stability. Source, sink, basin of attraction. Phase plane analysis of simple systems, homoclinic and heteroclinic orbits, hyperbolicity, statement of Hartmann-Grobman theorem and stable manifold theorem and their implications. (6L)

Liapunov function and Liapunov theorem. Periodic solutions, limit cycles and their stability concepts. Statement of Lienard's theorem and its application to vander Pol equation, Poineare-Bendixsom theorem (statement and applications only), structural stability and bifurcation through examples of saddle-node, pitchfork and Hopf bifurcations. (7L)

References :

1. *P. Glendinning* – Stability, Instability and Chaos *(Cambridge University Press 1994).*

2. *Strogartz* – Non-linear Dynamics

3. *M. W. Hirsch & S. Smale* – Differential Equations, Dynamical Systems and Linear Algebra *(Academic Press 1974)*

4. *L. Perko* – Differential Equations and Dynamical Systems *(Springer – 1991)*

5. *Arnold* – Ordinary Differential Equations

Paper – MAS304
(Special Paper-I)
Total Lectures: 65 Marks: 50

A: Viscous Flows, Boundary Layer Theory and Magneto Hydrodynamics-I

Viscous Flows

Some exact solutions of Navier – Stokes' Equations: the flow due to suddenly accelerated plane wall; the flow near an oscillating plane wall; plane stagnation point flow (Hiemenz flow); the flow near a rotating disk; Hele-shaw flow; Bodewadt flow. (18L)

Navier-Stokes equations in non-dimensional form; Reynolds number. Creeping motion; hydrodynamical theory of lubrication; Stokes's flow past a sphere and a cylinder : Stokes paradox; Oseen approximation,Oseen's solution for a sphere. (18L)

Laminar Boundary Layer Theory

Concept of boundary layer : Prandtl's assumptions. Two dimensional B.L. Equations for flow over a plane wall : Boundary layer on a flat plate ; Blasius-Topfer solution, 'Similar solutions' of the B. L. equations : B. L. flow past a wedge; B. L. flow along the wall of a convergent channel; B. L. flow past a circular cylinder; (20L)

Separation of boundary layer.

The spread of a jet :

(i) *plane free jet (two-dimensional jet),*

(ii) *circular jet (axisymmetric jet).*

Prandtl-Mises transformation :

Karman momentum integral equation. Karman – Pohlhausen method : simple applications. (9L)

References:

1. *S. W. Yuan: Foundations of Fluid Mechanics, PHI, 1969*

2. *H. Schlichting: Boundary Layer Theory, Mc Graw-Hill Book Comp., 2004.*

3. *L.D. Landau and E. M. Lifshitz: Fluid Mechanics, Pergamon Press, 1959.*

4. *J. L. Bansal: Viscous Fluid Dynamics, 2003.*

5. *J. A. Shercliff: A text Book of Magnetrohydrodynamics*

6. *V. C. A. Ferraro and C. Plumpton: An Introduction to magneto fluid Mechanics, Oxford Univ. Press, 1961.*

B: Elasticity-I

1. *Generalised Hooke's law Orthotropic and transversely isotropic media. Stress-strain relations in isotropic elastic solid. (5L)*

2. *Saint-Venant's semi-inverse method of solution (Statement). Formulation of torsion problem and the equations satisfied by the torsion function and the boundary condition. Formulation of torsion problem as an internal Neumann problem,. Dirichlet's problem and Poisson's problem. Prandtl's stress function. shearing stress in torsion problem.*

Solution of torsion problem for simple sections Method of sol. of torsion problem by conformal mapping. (30L)

3. *Flexure problem : Reduction of flexure problem to Neumann problem. Solution of flexure problem for simple sections. (10L)*

4. *Potential energy of deformation. Reciprocal theorem of Betti and Rayleigh. Theorem of min. Potential energy. (10L)*

5. *Plane problem : plane strain, plane stress, generalised plane stress. Basic equations. Airy's stress function. Solution in terms of complex analytic function. (10L)*

References :

1. *Y. A. Amenzade* – Theory of Elasticity *(MIR Pub.)*

2. *A. E. H. Love* – A treatise on the Mathematical Theory of Elasticity, CUP, 1963.

3. *I. S. Sokolnikoff* – Mathematical Theory of Elasticity, Tata Mc Graw Hill Co., 1977.

4. *W. Nowacki* – Thermoelasticity *(Addison Wesley)*

5. *Y. C. Fung- Foundations of Solid Mechanics, PHI, 1965.*

6. *S. Timoshenk and N. Goodies, Theory of Elasticity, Mc Grwa Hill Co., 1970.*

7. *N. I. Muskhelishvili- Some Basic Problems of the mathematical theory of Elasticity, P. Noordhoff Ltd., 1963.*

C: Elasticity and Theoretical Seismology-I
Elasticity

Generalised Hooke's law. Transversely isotropic media. Stress-strain relations in isotropic media. (10L)
Saint-Venant's semi-inverse method of solution. Torsion problem. Equation satisfied by torsion function and the boundary condition. Prandtl's stress function. Max shearing stress. Solution of torsion problem for simple sections. (20L)
Flexure problem. Differential Equation and the boundary condition. Solution for simple sections.
Potential energy of deformation. Reciprocal theorem of Betti and Rayleigh. Theorem of minimum potential energy. (20L)
Plane strain, plane stress. Basic equations. Airy's stress function. Thermo-elasticity : Thermal stress. Stress-strain relation in Thermo-elasticity. (15L)

References :

1. *Y. A. Amenzade* – Theory of Elasticity *(MIR Pub.)*

2. *A. E. H. Love* – A treatise on the Mathematical Theory of Elasticity, CUP, 1963.
3. *I. S. Sokolnikoff* – Mathematical Theory of Elasticity, Tata Mc Graw Hill Co., 1977.
4. *W. Nowacki* – Thermoelasticity *(Addison Wesley)*
5. *Y. C. Fung- Foundations of Solid Mechanics, PHI, 1965.*
6. *S. Timoshenk and N. Goodies, Theory of Elasticity, Mc Grwa Hill Co., 1970.*
7. *N. I. Muskhelishvili- Some Basic Problems of the mathematical theory of Elasticity, P. Noordhoff Ltd., 1963.*

D: Applied Functional Analysis-I

Review of basic properties of Hilbert spaces. Convex programming-support functional of a convex set. Minkowski functional. Separation Theorem. Kuhn-Tucker Theorem. Minimax theorem. Farkas theorem. (20L)
Spectral theory of operators. Spectral Theory of compact operations. Operators on a separable Hilbert space. Krein factorization theorem for continuous kernels and its consequences. L_2 spaces over Hilbert spaces. (30L)
Multilinear forms. Analyticity Theorems. Non-linear Volterra operators. (15L)

References :

5. A. V. Balakrishnan- *Applied Functional Analysis,* Springer-Verlag.
6. Dunford and Schwartz-*Linear operators,* vol. 1 & 11.
7. S. G. Krein-Linear Differential Equations in a Banach space.
8. K. Yosida-Functional Analysis.

Paper – MAS305
(Special Paper-II)
Total Lectures : 65 (Marks – 50)
A: Quantum Mechanics -I

1. Transformation Theory : Adjoint operator, Hermitian operator, Projection operator, Degeneracy, Unitary transformation, Matrix representation of wave functions and operators, Change of basis, Transformation of matrix elements, Dirac's Bra and Ket

notation, Completeness and normalization of eigen functions, Common set of eigen functions of two operators, Compatibility of observables. (15L)

2. *Symmetries and Invariance : Angular momentum eigenvalues and eigenfunctions, Spin, Addition of two angular momenta, Rotation groups, Identical particles, Pauli exclusion principle, Invariance and conservation theorems. (15L)*

3. *Relativistic Kinematics : Kelin-Gordon equation, Dirac equation for a free particle and its Lorentz covariance, Hole theory and positron, Electron spin and magnetic moment. (15L)*

4. *Approximation Methods (time-independent)*

Rayleigh-Schrödinger perturbation method, An harmonic oscillator, Stark effect in hydrogen atom, Zeeman effect, Ground state energy of helium atom. (10L)

5. *Elements of Second Quantization of A System : Creation and Annihilation operator, Commutation and Anti-commutation rules, Relation with Statistics - Bosons and Fermions. (10L)*

References :

1. *A. Messiah* – Quantum Mechanics, *Vol. I & II (North – Holland Pub. Co., 1962).*

2. *B. H. Bransden & C. J. Joachain* – Introduction to Quantum Mechanics *(Oxford University Press, 1989).*

3. *P. G. Burke* – Potential Scattering in Atomic Physics *(Plenum Press, New York, 1977)*

4. *C. J. Joachain* – Quantum Collision Theory *(North-Holland Pub. Co., 1975)*

5. *B. H. Bransden* – Atomic Collision Theory *(W. A. Benjamin Inc., N. Y., 1970)*

6. *S. Geltman* – Topics in Atomic Collision Theory *(Academic Press. 1969)*

7. *T. Y. Wu and T. Olmura* – Quantum Theory of Scattering *(Prentice Hall, New Jersey, 1962)*

8. *N. F. Mott & H. S. W. Massey* – Theory of Atomic collisions *(3rd ed.), (Clarendon Press, Oxford, 1965)*

9. *M. L. Goldberger & K. M. Watson* – Collision Theory *(Wiley, N. Y., 1964)*

10. *R. G. Newton* – Scattering Theory of Waves and Particles *(McGraw – Hill, 1966)*

B: Advanced Operations Research-I

Non-linear Optimization : Local and global minima and maxima, convex functions and their properties, Method of Lagrange multiplier. (8L)

60

Optimality in absence of differentiability, Slater constraint qualification, Karlin's constraint qualification, Kuhn-Tuckers Saddle point optimality conditions, Optimality criterion with differentiability and convexity, separation theorems, Kuhn-Tuckers sufficient optimality theorem. (10L)

Unconstrained Optimization : Search method : Fibonacci search, Golden Section search; Gradient Methods : Steepest descent Quasi-Newton's method, Davidon-Fletecher-Powell method, Conjugate direction method (Fletecher-Reeves method). (15L)

Optimality conditions : Kuhn-Tucker conditions – non negative constraints (6L)

Quadratic Programming : Wolfe's Modified Simplex method, Beale's method (8L)

Separable convex programming, Separable Programming Algorithm. (6L)
Network Flow : Max-flow min-cut theorem, Generalized Max flow min-cut theorem, Linear Programming interpretation of Max-flow min-cut theorem, Minimum cost flows, Minflow max-cut theorem. (12L)

References:

13. G. Hadly – *Non-Linear and Dynamic Programming,* Addision –Wesley, Reading Mass.

14. G. Hadly – Linear Programming, Narosa Publishing House.

15. S. S. Rao –*Optimization theory and Applications,* Wiley Eastern Ltd., New Delhi.

16. O. L. Mangasarian – *Non-Linear Programming,* McGraw Hill, New York.

17. Luenberger – *Introduction to Linear and Non-Linear Programming*

18. S. Dano – *Non-Linear and Dynamic Programming*

19. H. A. Taha – *Operations Research – An Introduction,* Macmillan Publishing Co., Inc., New York.

20. Swarup, Gupta & Manmohan – *Operations Research,* Sultan Chand & Sons, New Delhi.

21. *N.S. Kambo- Mathematical Programming Techniques*, Affiliated East-West Press Pvt. Ltd., New Delhi

22. M. C. Joshi and K.M. Moudgalya, Optimization theory and Practice, Narosa Publishing House, New Delhi

23. C.R. Bector, S. Chandra and J. Dutta, Principles of optimization Theory, Narosa Publishing House, New Delhi

24. M. A. Bhatti, Practical Optimization Methods, Springer -Verlag

C: Inviscid Compressible Flows and Turbulence-I

61

Basic thermodynamics; Equations of state; Polytropic gases. Euler's equations of Motion; conservation of energy. Circulation theorem; Propagation of a small disturbance : Sound waves. Steady isentropic motion : Bernoulli's eqn. Subsonic and supersonic flows. Irrotational flow : velocity potential; Bernoulli's eqn. for unsteady flow. Stream function for steady two-dimensional motion. Steady flows through stream tubes, De Laval Nozzle. (30L)
Method of characteristics, unsteady one-dimensional flow. Normal and oblique shock relations; shock polar diagram. (10L)
Exact solution for two-dimensional steady motions : Radial flow : vortex flow, Prandtl-Mayer flows. Hodograph method : Molenbroek transformation; Legendre transformation; Solution of Chaplygine's equation. Limit lines; Linearization by the method of small perturbation : Prandtl-Glauert transformation. Subsonic and supersonic flow past thin bodies. Rayleigh-Janzen's method for flow past a circular cylinder. (25L)

References:

1. S. W. Yuam: Foundations of Fluid Mechanics, PHI, 1969

2. H. Schlichting: Boundary Layer Theory, Mc Graw-Hill Book Comp., 2004.

3. *L.D. Landau and E. M. Lifshitz: Fluid Mechanics, Pergamon Press, 1959.*

4. *J. L. Bansal: Viscous Fluid Dynamics, 2003.*

5. *J. A. Shercliff: A text Book of Magnetrohydrodynamics*

6. *V. C. A. Ferraro and C. Plumpton: An Introduction to magneto fluid Mechanics, Oxford Univ. Press, 1961.*

D: Computational Fluid Dynamics-I

Finite Difference methods : Solution of O.D.E., The method of factoriazation, iterative methods, upwind corrected schemes, Hermitian method. Solution of a one-dimensional linear parabolic equation; Noncentered schemes, Leapfrog Dufort-Frankel scheme, Solution of one-dimensional Non-linear parabolic and hyperbolic equations, Explicit and Implicit methods, The ADI method, Explicit splitting method for two dimensional equation. (40L)

Finite Element Methods : Variational formulation of operator equations and Galerkin's method, the construction of the finite elements, convergence rates for F. E. M., Stability of F. E. M., Elementary ideas of Finite volume method, Spectral method. Some simple applications of Fluid Dynamics Problems. (25L)

References:

1. *Peter Linz – Theoretical Numerical Analysis, An Introduction To Advance Technique (John Wiley & Sons.)*

2. *R. Peyret and T. D. Taylor – Computational Methods for fluid Flow (Springer – Verlag)*

3. *P. Wesseling – Principles of Computational Fluid Dynamics (Springer-Verlag, 2000)*

4. *J. D. Anderson – Computational Fluid Dynamics : The Basics With Applications (McGraw-Hill,1998)*

5. *C. A. J. Fletcher* – Computational Techniques for Fluid Dynamics, *Vol. I and II (Springer-Verlag).*

6. *Dale A. Anderson, John C. Tanehill, Richard H. Pletcher* – Computational Fluid Mechanics and Heat Transfer *(Hemisphere Publishing Corporation)*

SEMESTER-IV

APPLIED MATHEMATICS STREAM

Paper – MAG401

(Continuum Mechanics-III)

Total Lectures : 65 (Marks – 50)

Inviscid incompressible fluid : Definitions, Constitutive equation for inviscid fluid. Euler's equation of motion – vector invariant form. Steady motion – Bernoulli's equation and other consequences. Euler's momentum theorem; D'Alembert's paradox Helmholtz equation for vorticity. Circulation. Kelvin's theorem on circulation. Irrotational motion, velocity potential, acyclic irrotational motion. Permanence of irrotational motion. Some properties of acyclic irrotatioal motion (using Greeen's theorem) and Uniqueness theorems. General equation for impulsive motion and properties of motion under surface impulse. Kelvin's minimum energy theorem. Three dimensional source, sink and doublet (definitions only). Two dimensional motion. Stream function. Complex potential. Circular line vortex. Complex potentials for line source (sink), line doublet and line vortex. Circle theorem. Method of Images. Blasius theorem for thrust on an obstacle-applications to circular cylindrical boundary.

Flow past a circular cylinder with circularion; Kutta-Joukowski's lift formula. Axi-symmetic motion.

63

Stokes stream function. Vortex motion-vortex surface, vortex tube and vortex filament. Fundamental properties (Helmholtz properties) of vortex motion. Velocity field due to a distribution of vorticity. Velocity field due to a closed vortex filament. (40L)

Gravity waves (water waves) surface condition, Cisotti's equation for complex polential of small height gravity waves. Progressive waves – cases of deep and shallow water. Stationary waves – possible wavelengths in a rectangular tank. Paths of particles for different waves. Energy for progressive waves and stationary wave. Group velocity and its dynamical significance. (10L)

Linearly viscous incompressible fluid:-

Constitutive equations (stress-rate-of strain relations) for linearly viscous fluid. Navier-Stokes equations-vector invariant form. Boundary conditions. Helmholtz equation for diffusion of vorticity.Dissipation of energy. Non-dimensional form of N.S. equations. Principle of similitude. Reynolds number Simple exact solutions of N-S equations: Parallel flow, Generalized Couette flow-plane. Plane Poiseuille flow and simple Couette flow. Hagen-Poiseuille flow through a circular pipe. Viscometric flow-Couette circular motion. (15L)

References :

1. Leigh, D. C. – *Non-Linear Continuum Mechanics* (MacGraw-Hill)

2. Truesdell, C – *Continuum Mechanics*

3. Chung, T. J. – *Contunuum Mechanics* (Prentice-Hall)

4. Truesdell, C and Nol, W. – *Encyclopaedia of Physics.* Vol. III/3, 1965 (Ed. S. Flugge)

5. Sokolnikoff, I. S. – *Mathematical Theory of Elasticity*

6. Miline – Thomson, L. M.- *Theoretical Hydrodynamics*

7. Pai, S. L. – *Viscous Flow Theory*

8. Schilichting, H. – *Boundary Layer Theory*

9. Eriengen, A. C. – *Non-linear Theory of Continuous Media* (MacGraw-Hill)

10. F. Chorlton – *A Text Book of Fluid Mechanics*

11. Kolin, Kebel & Roze – *Theoretical Hydromechanics*

12. Besand & Ramsey – *Fluid Mechanics*

13. J. Bansal – *Viscous Flow Theory*

14. Leigh, D. C. – *Non-Linear Continuum Mechanics* (MacGraw-Hill)

15. Truesdell, C – *Continuum Mechanics*

16. Chung, T. J. – *Contunuum Mechanics* (Prentice-Hall)

17. Truesdell, C and Nol, W. – *Encyclopaedia of Physics.* Vol. III/3, 1965 (Ed. S. Flugge)

18. Sokolnikoff, I. S. – *Mathematical Theory of Elasticity*

19. Miline – Thomson, L. M.- *Theoretical Hydrodynamics*

20. Pai, S. L. – *Viscous Flow Theory*

21. Schilichting, H. – *Boundary Layer Theory*

Paper – MAG402
(Elements of Quantum Mechanics, Chaos and Fractals)
Unit-1
Elements of Quantum Mechanics
Total Lectures : 40 (Marks – 30)

1. *Origin of the Quantum Theory :*
Black-body radiation, Inadequacy of classical theory, The old quantum theory, Bohr-Sommerfeld theory, Atomic Spectra, Photoelectric effect and Compton effect, Matter waves, Wave-particle duality, Electron diffraction experiment. (15L)

2. *Basic Concepts :*
Wave function of a free particle, Uncertainty and Complementarity principles, Gedanken experiments, wave packet, Schrödinger wave equation, Statistical interpretation of the wave function, Formal solution of the Schrödinger equation. (10L)

3. Simple Applications (exact solutions) :
One dimensional potential step, Potential barrier, Square-well potential, Linear harmonic oscillator, Three-dimensional box potential, Spherically symmetric potential, Hydrogen atom bound-state problems. (10L)

4. Dynamical Variables and Operators :
Operators corresponding to physical observables, Expectation values of observables, The virial theorem, Eigenfunction and eigenvalues of operators, Discrete and continuous spectra, Commutativity of operators, Heisenberg's uncertainty relations, The minimum uncertainty product, Heisenberg's equation of motion for operators. (5L)

References :

1. *Heisenberg* – The Physical Principles of the Quantum Theory *[Dover Pub., 1930]*

2. *P. A. M. Dirac* – The Principles of Quantum Mechanics *[Oxford University Press, 1981]*

3. *F. Mandl* – Quantum Mechanics *[Butterworths Sci. Pub., London, 1957]*

4. *P. T. Mathews* – Introduction to Quantum Mechanics *[MacGraw Hill, 1963]*

5. *L. I. Schiff* – Quantum Mechanics *[MacGraw Hill, 1968]*

Unit-2
Chaos and Fractals
Total Lectures : 25 (Marks – 20)
Chaos and Fractals : *Examples, graphical analysis, orbits, phase diagrams fixed and periodic points stable and unstable sets smooth maps and conditions for stable and unstable periodic points hyperbolicity. (10)*
SDIC, topological transivity (mixing) and Devancey's definition of chaos, binary decimal representation of numbers and saw tooth map. One parameter family of maps and bifurcations (through examples only) topological conjugacy, Logistic map, period doubling route to chaos (10L)
Cantor sets, examples of fractals, definitions of topological and capacity dimensions,

Horse shoe and the theorem : "period 3 implies chaos". (5L)

References :

1. *P. Glendinning* – Stability, Instability and Chaos *(Cambridge University Press 1994).*

2. *Strogartz* – Non-linear Dynamics

3. *R. L. Devaney* – A First Course In Chaotic Dynamical Systems

4. *R. A. Holmgren* - A First Course In Discrete Dynamical Systems *(Springer, 1991)*

5. *R. L. Devaney* – An Introduction To Chaotic Dynamical System *(Addison-Wesley 1987)*

6. *H. O. Peitgen* – Fractals and Chaos.

Paper – MAS403
(Special Paper-III)
Total Lectures: 65 Marks: 50
A: Viscous Flows, Boundary Layer Theory & Magneto-Hydrodynamics-II

Electromagnetic equations for moving media, Ohm's law including Hall current, Lorentz force. MHD approximations. Stress-tensor formulation of Lorentz force; frozen-in-magnetic field. Alfven's Theorem; Alfven waves. Equations of motion and induction; their nondimensional forms; dimensionless parameters, Lundquist's criterion. Energy equation : Viscous and Joule dissipation, Poynting theorem. Boundary conditions. (25L)

Steady viscous incompressible flows : unidirectional flow under a transverse magnetic field : decoupling of MHD equations. Hartmann flow; Couette flow. Flow through a rectangular duct. Unsteady incompressible flows. Rayleigh's problem. MHD waves : propagation of small disturbances; plane waves; Reflection and transmission of plane harmonic waves; existence of finite amplitude MHD waves. Alfven waves with ohmic damping; Skin effect. (25L)

Magnetohydrostatics; equilibrium-configurations, Pinch effect, force-free fields, non-existence of force-free field of finite extent. General solution for a force-free field, special cases.

Dynamo problem, Cowling's theorem, Ferraro's law of isorotation. (15L)

References:

1. *S. W. Yuan: Foundations of Fluid Mechanics, PHI, 1969*

2. *H. Schlichting: Boundary Layer Theory, Mc Graw-Hill Book Comp., 2004.*

3. *L.D. Landau and E. M. Lifshitz: Fluid Mechanics, Pergamon Press, 1959.*

4. *J. L. Bansal: Viscous Fluid Dynamics, 2003.*

5. *J. A. Shercliff: A text Book of Magnetrohydrodynamics*

6. *V. C. A. Ferraro and C. Plumpton: An Introduction to magneto fluid Mechanics, Oxford Univ. Press, 1961.*

B: Elasticity-II

Vibration problems : Longitudinal vibration of thin rods, Torsional vibration of a solid circular cylinder and a solid sphere. Free Rayleigh and Love waves. (15L)

Thermoelasticity : Stress-strain relations in Thermo elasticity. Reduction of statistical thermo-elastic problem to a problem of *isothermal elasticity. Basic equations in dynamic thermo elasticity. Coupling of strain and temperature fields. (30L)*

Magneto-elasticity : Interaction between mechanical and magnetic field. Basic equations Linearisation of the equations. (20L)

References :

1. *Y. A. Amenzade* – Theory of Elasticity *(MIR Pub.)*

2. *A. E. H. Love* – A treatise on the Mathematical Theory of Elasticity, CUP, 1963.

3. *I. S. Sokolnikoff* – Mathematical Theory of Elasticity, Tata Mc Graw Hill Co., 1977.

4. *W. Nowacki* – Thermoelasticity *(Addison Wesley)*

5. *Y. C. Fung- Foundations of Solid Mechanics, PHI, 1965.*

6. *S. Timoshenk and N. Goodies, Theory of Elasticity, Mc Grwa Hill Co., 1970.*

7. *N. I. Muskhelishvili- Some Basic Problems of the mathematical theory of Elasticity, P. Noordhoff Ltd., 1963.*

C: Elasticity and Theoretical Seismology-II

Theoretical Seismology

Theory of elastic waves; Motion of a surface of discontinuity – kinematical condition and dynamical conditions. Kirchoff's solution of inhomogeneous wave equation. (20L)

Reflection and refraction of elastic body waves. (10L)

Surface waves : Rayleigh, Love and Stonely waves (10L)

Dispersion and Group Velocity of elastic body waves. (10L)

Some problems : Application of pressure and twist on the walls of a spherical cavity in an elastic medium. (10L)

Line source and point source on the surface of a semi-infinite elastic medium. (5L)

References :

1. *Y. A. Amenzade* – Theory of Elasticity *(MIR Pub.)*

2. *A. E. H. Love* – A treatise on the Mathematical Theory of Elasticity, CUP, 1963.

3. *I. S. Sokolnikoff* – Mathematical Theory of Elasticity, Tata Mc Graw Hill Co., 1977.

4. *W. Nowacki* – Thermoelasticity *(Addison Wesley)*

5. *Y. C. Fung- Foundations of Solid Mechanics, PHI, 1965.*

6. *S. Timoshenk and N. Goodies, Theory of Elasticity, Mc Grwa Hill Co., 1970.*

7. *N. I. Muskhelishvili- Some Basic Problems of the mathematical theory of Elasticity, P. Noordhoff Ltd., 1963.*

8. *E. Savarensky- Seismic waves, MIR Pub.*

9. *B.L. N. Kennett, Seismic wave propagation in Stratified Media, CUP*

10. *K.E. Bullen, An Introduction to the theory of Seismology, CUP.*

11. *Arinben-Menahem Sarva Jit Singh- Seismic waves and Sources, Springer-Verlag.*

D: Applied Functional Analysis-II

Semigroups of linear operators-general properties of semigroups. Generation of semigroups. Dissipative semigroups. Compact semigroups. (20L)

Holomorphic semigroups, Elementary examples of semigroups. Extension. Differential Equations. Cauchy Problem, Controllability. State reduction. Ovservability. Stability and stabilizability. Evolution equations. (30L)

Optimal Control Theory-Linear quadratic regulator problem. The same problem with infinite time interval. Hard constraints. Final value control. Time optimal control problems. (15L)

References :

1. A. V. Balakrishnan- *Applied Functional Analysis,* Springer-Verlag.

2. Dunford and Schwartz-*Linear operators,* vol. 1 & 11.

3. S. G. Krein-*Linear Differential Equations in a Banach space.*

4. K. Yosida-*Functional Analysis.*

Paper – MAS404

(Special Paper-IV)

Total Lectures : 65 (Marks – 50)

A: Quantum Mechanics -II

Collision Theory : Basic concepts, Cross sections, Laboratory and center-of-mass coordinates, Rutherford scattering, Quantum mechanical formulation – time independent and time-dependent, Scattering of a particle by a short-range potential, Scattering by Coulomb potential, Scattering by screened Coulomb field, Scattering by complex potential. (15L)

Integral Equation Formulation: Lippmann-Schwinger integral equation and its formal solutions, Integral representation of the scattering amplitude, Convergence of the Born Series, Validity of Born approximation, Transition probabilities and cross sections. (12L)

Semi-Classical Approximations : WKB approximation, Eikonal approximation. (8L)

69

Variational Principles in the Theory of Collisions : General formulation of the variational principle, Hulthen, Kohn-Hulthen and Schwinger variational methods, Determination of Phase shifts, Scattering length and scattering amplitude for central force problems, Bound (minimum) principles. (20L)

Analytic Properties of Scattering Amplitude : Jost function, Scattering matrix, Bound states and resonances, Levinson theorem, Dispersion relations, Effective range theory. (10L)

References :

1. *A. Messiah* – Quantum Mechanics, *Vol. I & II (North – Holland Pub. Co., 1962).*

2. *B. H. Bransden & C. J. Joachain* – Introduction to Quantum Mechanics *(Oxford University Press, 1989).*

3. *P. G. Burke* – Potential Scattering in Atomic Physics *(Plenum Press, New York, 1977)*

4. *C. J. Joachain* – Quantum Collision Theory *(North-Holland Pub. Co., 1975)*

5. *B. H. Bransden* – Atomic Collision Theory *(W. A. Benjamin Inc., N. Y., 1970)*

6. *S. Geltman* – Topics in Atomic Collision Theory *(Academic Press. 1969)*

7. *T. Y. Wu and T. Olmura* – Quantum Theory of Scattering *(Prentice Hall, New Jersey, 1962)*

8. *N. F. Mott & H. S. W. Massey* – Theory of Atomic collisions *(3rd ed.), (Clarendon Press, Oxford, 1965)*

9. *M. L. Goldberger & K. M. Watson* – Collision Theory *(Wiley, N. Y., 1964)*

10. *R. G. Newton* – Scattering Theory of Waves and Particles *(McGraw – Hill, 1966)*

B: Advanced Operations Research-II

Dynamic Programming : Characteristics of Dynamic Programming problems, Bellman's principle of optimality (Mathematical formulation)

Model –1 : Single additive constraint, multiplicative separable return,

Model – 2 : Single additive constraint, additively separable return,

Model – 3 : Single a multiplicative constraint, additively separable return,

Multistage decision process – Forward and Backward recursive approach, Dynamic Programming approach for solving linear and non-linear programming problems, Application – Single-item N-period deterministic inventory model. (25L)

70

Geometric Programming : Elementary properties of Geometric Programming and its applications. (8L)

Queuing Theory : Introduction, characteristic of Queuing systems, operating characteristics of Queuing system. Probability distribution in Queuing systems. Classification of Queuing models. Poisson and non-Poisson queuing models (32L)

References:

6 G. Hadly – *Non-Linear and Dynamic Programming*, Addision –Wesley, Reading Mass.

7 S. Dano – *Non-Linear and Dynamic Programming*

8 H. A. Taha – *Operations Research – An Introduction*, Macmillan Publishing Co., Inc., New York..

9 Swarup, Gupta & Manmohan – *Operations Research*, Sultan Chand & Sons, New Delhi.

10 N.S. Kambo- *Mathematical Programming Techniques*, Affiliated East-West Press Pvt. Ltd., New Delhi

C: Inviscid Compressible Flows and Turbulence-II

Turbulence:

Introduction. Reynold's equations of mean motion for turbulent flow, Reynold's stress-tensor; eddy viscosity. Phenomenological theories. Mixing length Prandtl's momentum transfer theory, Taylor's vorticity transfer theory. Karman's similarity hypothesis. Velocity distribution in channel flow under constant pressure gradient. (30L)

Spread of turbulence: Mixing zone between two parallel flows, (two-dimensional) turbulent wake behind (i) symmetrical cylinder (ii) a row of parallel rods. Turbulent flow through smooth circular pipes; Seventh power velocity distribution law; turbulent boundary layer on a flat plate. (20L)

Statistical approach; Introductory concepts; double correlation between velocity components, longitudinal and lateral; correlations in homogeneous turbulence; Eulerian correlation with respect to time, Taylor's one-dimensional energy spectrum. Energy relations in turbulent flows. (15L)

References:

1. *S. W. Yuam: Foundations of Fluid Mechanics, PHI, 1969*

2. *H. Schlichting: Boundary Layer Theory, Mc Graw-Hill Book Comp., 2004.*

3. *L.D. Landau and E. M. Lifshitz: Fluid Mechanics, Pergamon Press, 1959.*

4. *J. L. Bansal: Viscous Fluid Dynamics, 2003.*

5. *J. A. Shercliff: A text Book of Magnetrohydrodynamics*

6. *V. C. A. Ferraro and C. Plumpton: An Introduction to magneto fluid Mechanics, Oxford Univ. Press, 1961.*

D: Computational Fluid Dynamics-II

Multigrid method, Conjugate – Gradient method. Incompressible Navier – Stokes (NS) equations – Boundary conditions, Spatial and temporal discretization on collocated and on staggered grids. Development of the MAC Method for NS equations, Implementation of boundary conditions. (35L)

Grid Generation by Algebraic mapping : One-dimensional stretching functions, Boundary – Filted Coordinate Systems : Elliptic Grid generation. Solution of Euler Equations in General Co-ordinates. Numerical Solution of NS Equations in General Co-ordinates. (30L)

References :

1. Peter Linz – *Theoretical Numerical Analysis, An Introduction To Advance Technique* (John Wiley & Sons.)

2. R. Peyret and T. D. Taylor – *Computational Methods for fluid Flow* (Springer – Verlag)

3. P. Wesseling – *Principles of Computational Fluid Dynamics* (Springer-Verlag, 2000)

4. J. D. Anderson – *Computational Fluid Dynamics : The Basics With Applications* (McGraw-Hill,1998)

5. C. A. J. Fletcher – *Computational Techniques for Fluid Dynamics,* Vol. I and II (Springer-Verlag).

6. Dale A. Anderson, John C. Tanehill, Richard H. Pletcher – *Computational Fluid Mechanics and Heat Transfer* (Hemisphere Publishing Corporation)

Paper – MAT405

Term Paper

Marks: 50

Term paper MAT405 is related with the special papers of the applied stream offered by the department in each session and the topic of the term paper will also be decided by the department in each session. However the mark distribution is 30 marks for written submission, 15 marks for seminar presentation and 5 marks for viva-voce.

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Dr. S. K. Kapoor

Ved Ratan